Ensemble Distribution for Immiscible Two-Phase Flow in Porous Media

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Aim:

A statistical description of two-phase flow through a porous medium from the microscale to the macroscale

- Compare results with network simulations
- Compare results with experiments

System: a 2D dynamic network model

In our analysis we use links which are all the same. For other networks one has links that differ. This must be accounted for.
Defining the ensemble distribution

The ensemble distribution is a \textit{time-independent} probability distribution of two fluids over the links in a representative elementary volume (REV) under steady-state conditions.
Defining the ensemble distribution

On the **individual link level**, the ensemble distribution gives the joint probability density, that this link has a given radius $r_{0,ij}$, saturation $s_{ij}$, and center-of-mass position for the non-wetting fluid $x_{b,ij}$.

If there are different links one adds a label to indicate that.
Postulating a general form of ensemble distribution

$$\Pi(x_{bi,j}, s_{ij}, r_{0,ij}) = \frac{\langle |q| \rangle}{|q_{ij}(x_{bi,j}, s_{ij}, r_{0,ij})|} f(x_{bi,j}, s_{ij}, r_{0,ij})$$

- $f(x_{bi,j}, s_{ij}, r_{0,ij})$ is normalized.
- It is important to realize that the above postulate is not more than a definition or $f$ !!!
- The distribution is inversely proportional to the volume flow $q_{ij}$ in the links. The slower a bubble moves in a tube, the longer is the time that it spends in that tube! This motivates definition.
- For different links a label must be added.
- In 1-D there is an analytical proof:

Testing $1/q$ dependence

The $1/q$ dependence is clearly not perfect !!!

Further testing the form of distribution

- The joint distribution $f(x_{b,ij}, s_{ij}, r_{0,ij})$ should be independent of the volume flow $q$.

- To verify this, we plotted $f$ for different ranges of volume flows, for $Ca = 0.01$ and global saturation $s = 0.20$. The histograms are qualitatively similar, but $f$ shows some dependence on $q$. 
(a) $1 \text{ mm}^3/\text{s} < q_{ij} < 2 \text{ mm}^3/\text{s}$

(b) $2 \text{ mm}^3/\text{s} < q_{ij} < 3 \text{ mm}^3/\text{s}$

(c) $3 \text{ mm}^3/\text{s} < q_{ij} < 4 \text{ mm}^3/\text{s}$

(d) $4 \text{ mm}^3/\text{s} < q_{ij} < 5 \text{ mm}^3/\text{s}$
Results

- Average absolute volume flow:
  \[ \langle |q| \rangle = \int_0^\infty dr_{0,ij} \int_0^\infty ds_{ij} \int_0^\infty dx_{b,ij} \prod(x_{b,ij}, s_{ij}, r_{0,ij}) \]
  \[ \times |q_{ij}(x_{b,ij}, s_{ij}, r_{0,ij})| \].

- The total absolute volume flow through a layer:
  \[ \langle |Q| \rangle = L \langle |q| \rangle , \]
  where \( L \) is the number of links in a layer.

Both results valid for arbitrary \( q \) dependence of \( f \) !!!
Results

The total absolute volume flow of the non-wetting, $|Q_n|$ fluid is

$$
\langle |Q_n| \rangle = L \int_0^\infty dr_{0,ij} \int_0^1 ds_{ij} \int_0^l dx_{b,ij} \\
\quad \times \prod (x_{b,ij}, s_{ij}, r_{0,ij}) s_{ij} \left| q_{ij} (x_{b,ij}, s_{ij}, r_{0,ij}) \right| \\
= L \langle |q| \rangle \int_0^\infty dr_{0,ij} \int_0^1 ds_{ij} \int_0^l dx_{b,ij} s_{ij} f(x_{b,ij}, s_{ij}, r_{0,ij}) \\
= L \langle |q| \rangle \langle s \rangle.
$$

Also true for arbitrary $q$ dependence of $f$ !!!

Results

The fractional flow $F$ of the non-wetting fluid is equal to the ensemble averaged saturation, which is the straight average of the link saturations.

$$F = \frac{\langle |Q_n| \rangle}{\langle |Q| \rangle} = \int_0^\infty dr_{0,ij} \int_0^1 ds_{ij} \int_0^l dx_{b,ij} s_{ij} f(x_{b,ij}, s_{ij}, r_{0,ij}) = \langle s \rangle$$
The fractional flow $F$ of the non-wetting fluid is not equal to the volume averaged saturation $s = \langle r_0^2 s \rangle / \langle r_0^2 \rangle$ which is measured.

Results

The viscous dissipation is the same, whether we compute it from the distribution function or from overall considerations.

Again equal for arbitrary $q$ dependence of $f$ !!!
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Co-authors

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Thank you for your attention!

Welcome to Trondheim