Ensemble Distribution for Immiscible Two-Phase Flow in Porous Media

Isha Savani[1], Dick Bedeaux[2], Signe Kjelstrup[2], Morten Vassvik[1], Santanu Sinha[3], Alex Hansen[1]

[1]Department of Physics and [2]Department of Chemistry

Norwegian University of Technology and Science, Trondheim, Norway

[3]Beijing Computational Science Research Center

Aim:

A statistical description of two-phase flow through a porous medium from the microscale to the macroscale

- Compare results with network simulations
- Compare results with experiments

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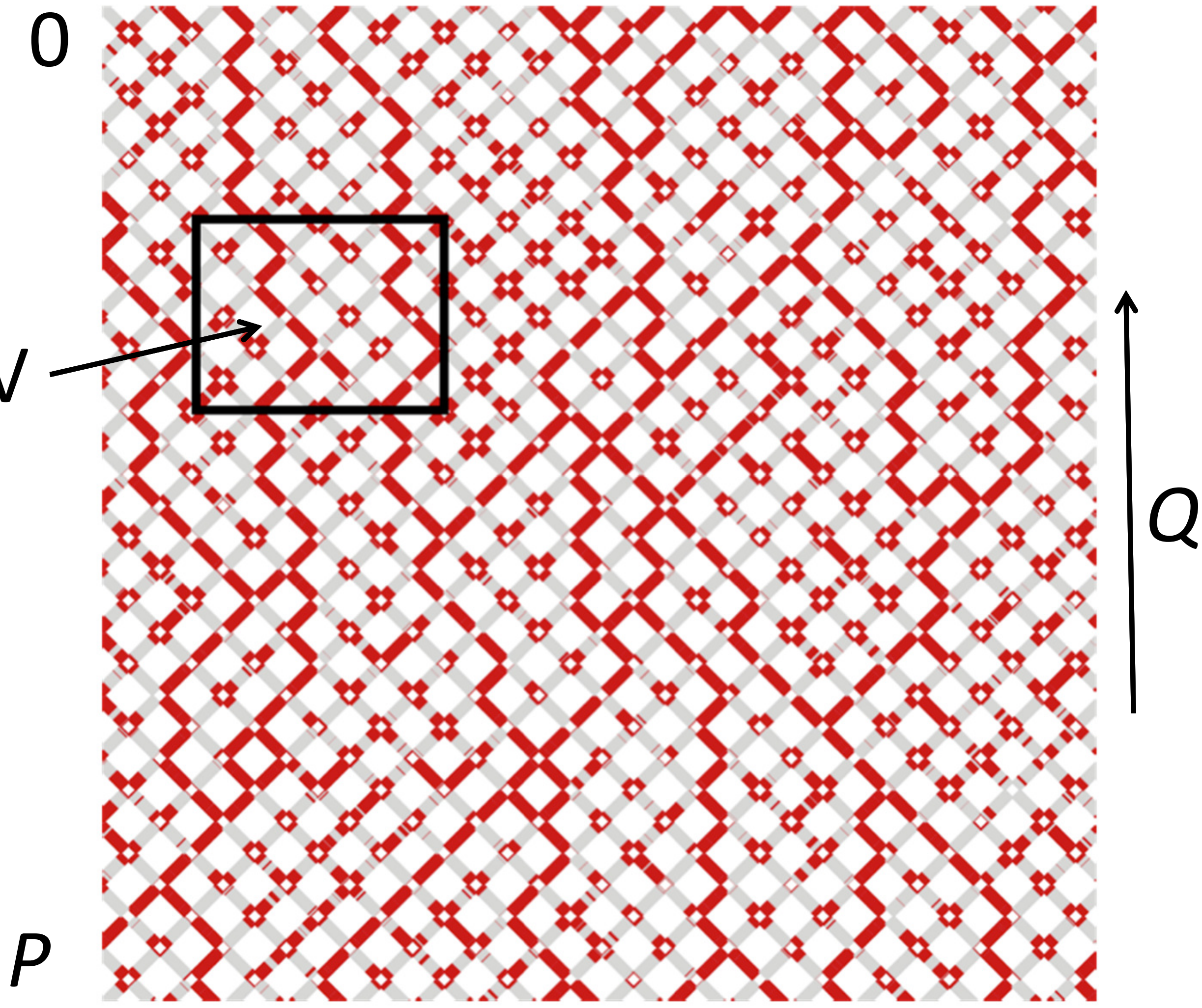
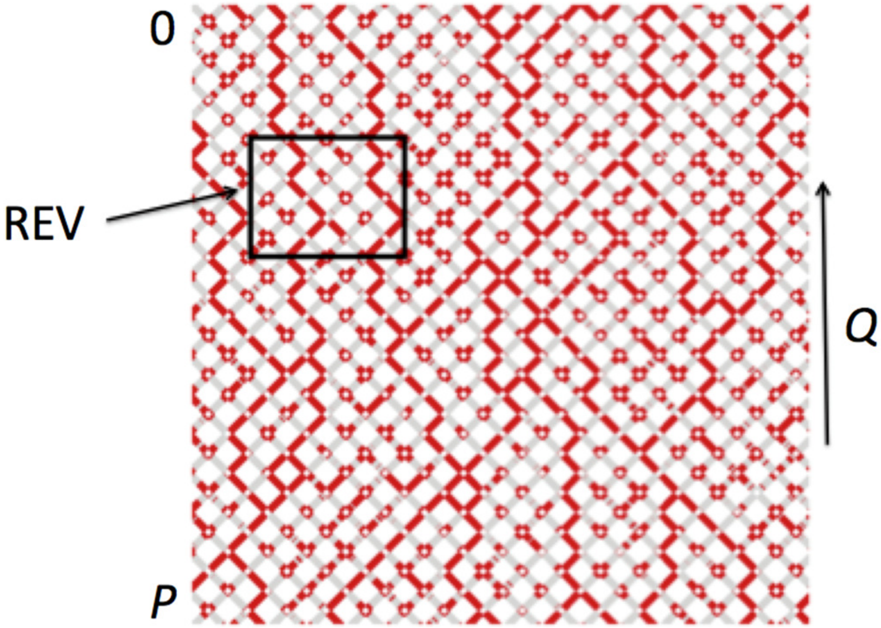
System: a 2D dynamic network model

In our analysis we use links which are all the same. For other networks one has links that differ. This must be accounted for.

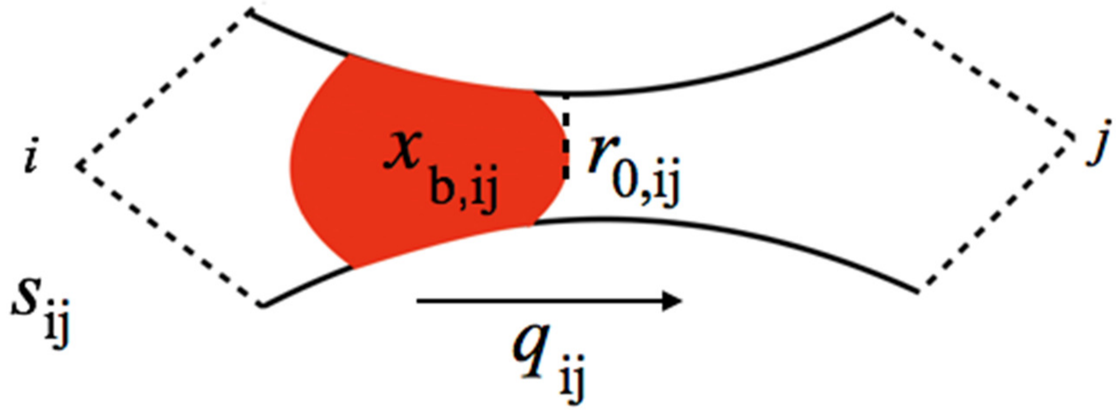


Defining the ensemble distribution



The ensemble distribution is a time-independent probability distribution of two fluids over the links in a representative elementary volume (REV) under steady-state conditions

Defining the ensemble distribution



On the individual link level, the ensemble distribution gives the joint probability density, that this link has a given radius $r_{0,ij}$, saturation s_{ij}, and center-of-mass position for the non-wetting fluid $x_{b,ij}$

If there are different links one adds a label to indicate that.

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Postulating a general form of ensemble distribution

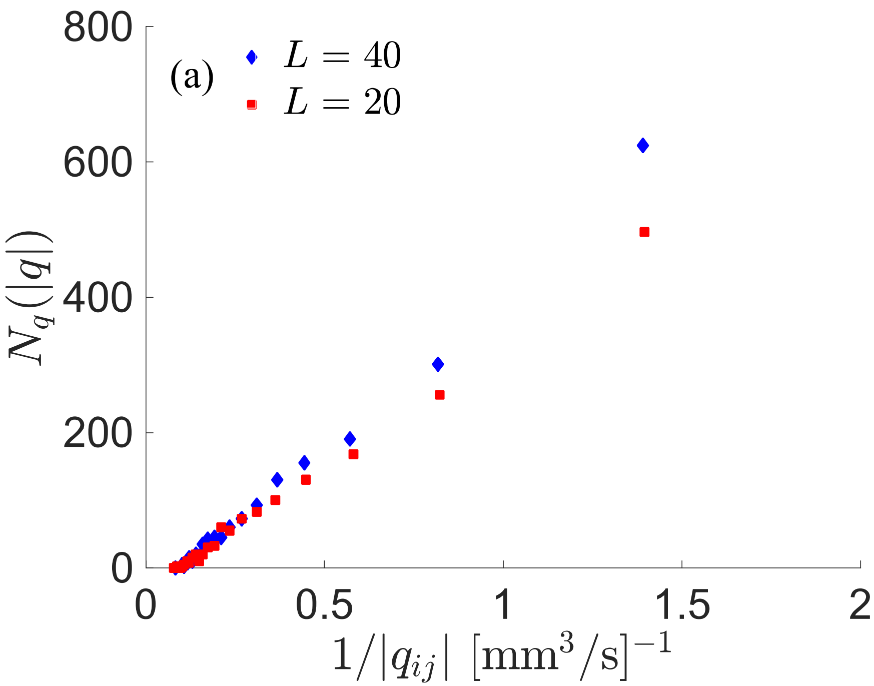
$$\Pi(x_{b,ij}, s_{ij}, r_{0,ij}) = \frac{\langle |q| \rangle}{|q_{ij}(x_{b,ij}, s_{ij}, r_{0,ij})|} f(x_{b,ij}, s_{ij}, r_{0,ij})$$

$$\text{for } 0 < x_b < l \text{ and } 0 < s_{ij} < 1$$

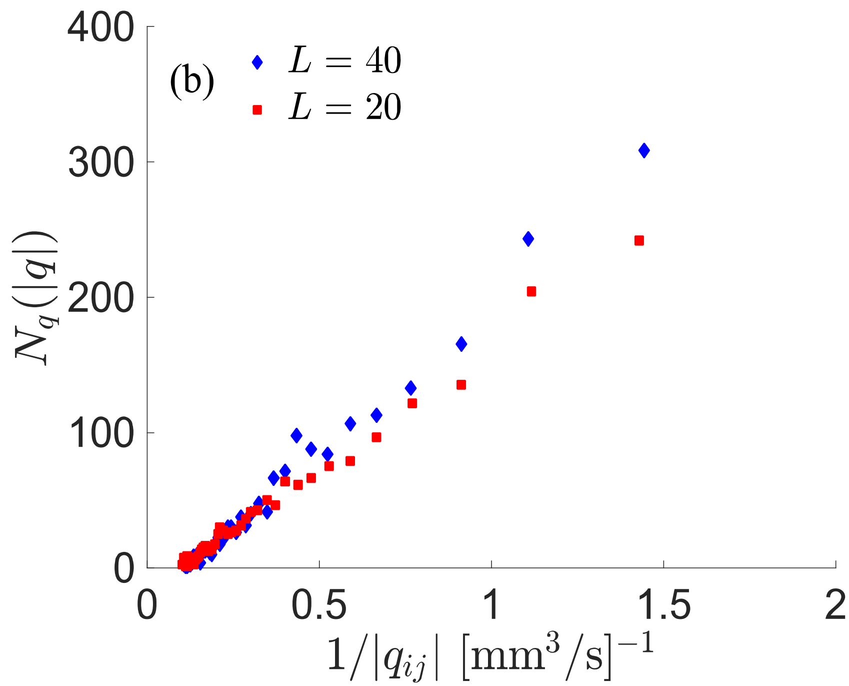
- $f(x_{b,ij}, s_{ij}, r_{0,ij})$ is normalized.
- It is important to realize that the above postulate is not more than a definition or *f* !!!
- The distribution is inversely proportional to the volume flow q_{ij} in the links. The slower a bubble moves in a tube, the longer is the time that it spends in that tube! This motivates definition.
- For different links a label must be added.
- In 1-D there is an analytical proof:

S Sinha, A Hansen, D Bedeaux, and S Kjelstrup, Phys. Rev. E **87** (2013) 025001.

Testing 1/q dependence



Histogram of link volume flow $N_q(|q|)$ for $0.3 < s_{ij} < 0.4$, $0.2 < r_{0,ij} < 0.3$ mm, $s = 0.3$ and Ca = 0.01.



Histogram of link volume flow $N_q(|q|)$ for $0.4 < s_{ij} < 0.5$, $0.2 < r_{0,ij} < 0.3$ mm, $s = 0.5$ and Ca = 0.01.

The 1/q dependence is clearly not perfect !!!

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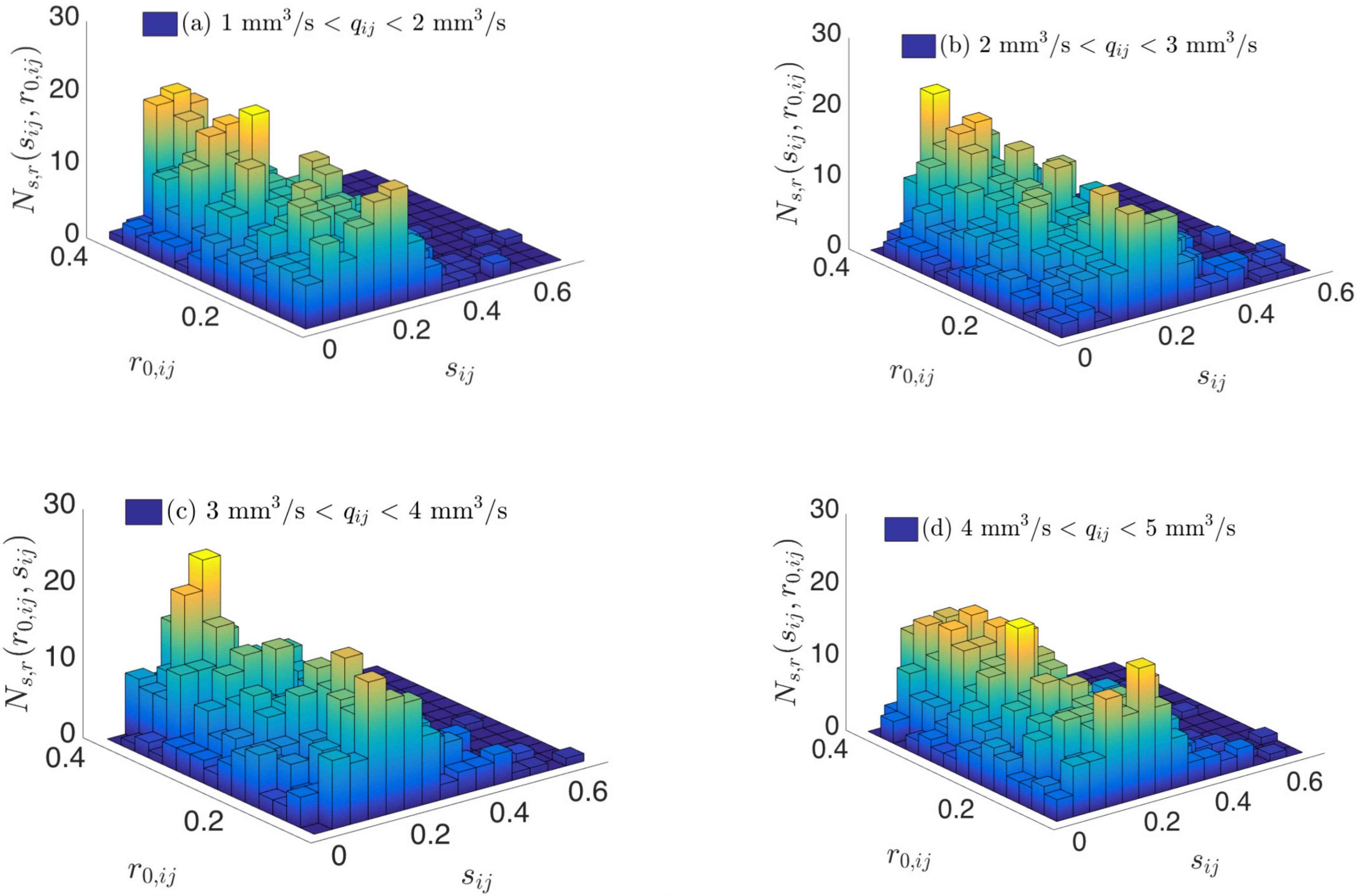
Further testing the form of distribution

- The joint distribution $f(x_{b,ij}, s_{ij}, r_{0,ij})$ should be independent of the volume flow *q*.

- To verify this, we plotted *f* for different ranges of volume flows, for Ca = 0.01 and global saturation s = 0.20. The histograms are qualitatively similar, but *f* shows some dependence on *q.*


(a) 1 mm^3/s < q_{ij} < 2 mm^3/s
$N_{s,r}(s_{ij}, r_{0,ij})$
30
20
10
0
0.4
0.2
$r_{0,ij}$
0
0.2
0.4
0.6
s_{ij}
(b) 2 mm^3/s < q_{ij} < 3 mm^3/s
$N_{s,r}(s_{ij}, r_{0,ij})$
30
20
10
0
0.4
0.2
$r_{0,ij}$
0
0.2
0.4
0.6
s_{ij}
(c) 3 mm^3/s < q_{ij} < 4 mm^3/s
$N_{s,r}(r_{0,ij}, s_{ij})$
30
20
10
0
0.4
0.2
$r_{0,ij}$
0
0.2
0.4
0.6
s_{ij}
(d) 4 mm^3/s < q_{ij} < 5 mm^3/s
$N_{s,r}(s_{ij}, r_{0,ij})$
30
20
10
0
0.4
0.2
$r_{0,ij}$
0
0.2
0.4
0.6
s_{ij}

Results

- Average absolute volume flow:

$$\langle |q| \rangle = \int_0^\infty dr_{0,ij} \int_0^\infty ds_{ij} \int_0^\infty dx_{b,ij} \Pi(x_{b,ij}, s_{ij}, r_{0,ij})$$
$$\times |q_{ij}(x_{b,ij}, s_{ij}, r_{0,ij})|.$$

- The total absolute volume flow through a layer:

$$\langle |Q| \rangle = L \langle |q| \rangle,$$

where *L* is the number of links in a layer.

Both results valid for arbitrary *q* dependence of *f* !!!

Results

The total absolute volume flow of the non-wetting, $\langle|Q_n|\rangle$ fluid is

$$
\begin{aligned}
\langle|Q_n|\rangle = L\int_0^\infty dr_{0,ij}\int_0^1 ds_{ij}\int_0^l dx_{b,ij} \\
\times\prod(x_{b,ij},s_{ij},r_{0,ij})s_{ij}\,|q_{ij}(x_{b,ij},s_{ij},r_{0,ij})| \\
= L\langle|q|\rangle\int_0^\infty dr_{0,ij}\int_0^1 ds_{ij}\int_0^l dx_{b,ij}s_{ij}f(x_{b,ij},s_{ij},r_{0,ij}) \\
= L\langle|q|\rangle\langle s\rangle.
\end{aligned}
$$

Also true for arbitrary *q* dependence of *f* !!!

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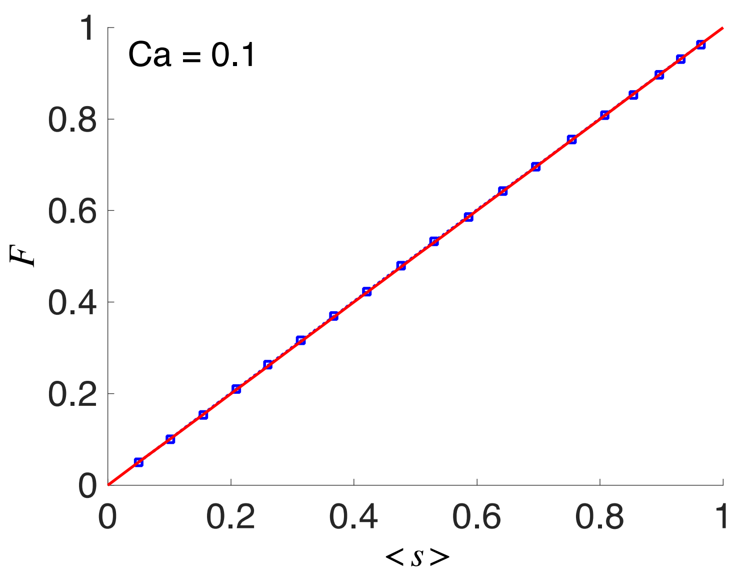
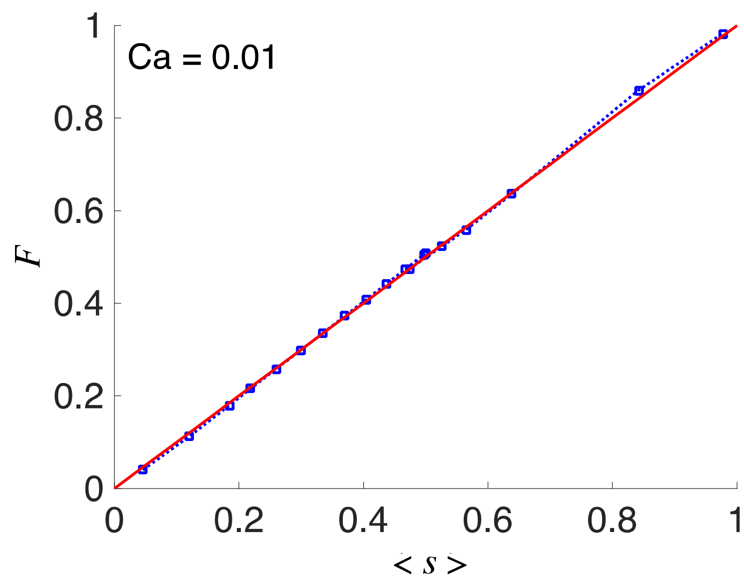
Results

The fractional flow *F* of the non-wetting fluid is equal to the ensemble averaged saturation, which is the straight average of the link saturations.

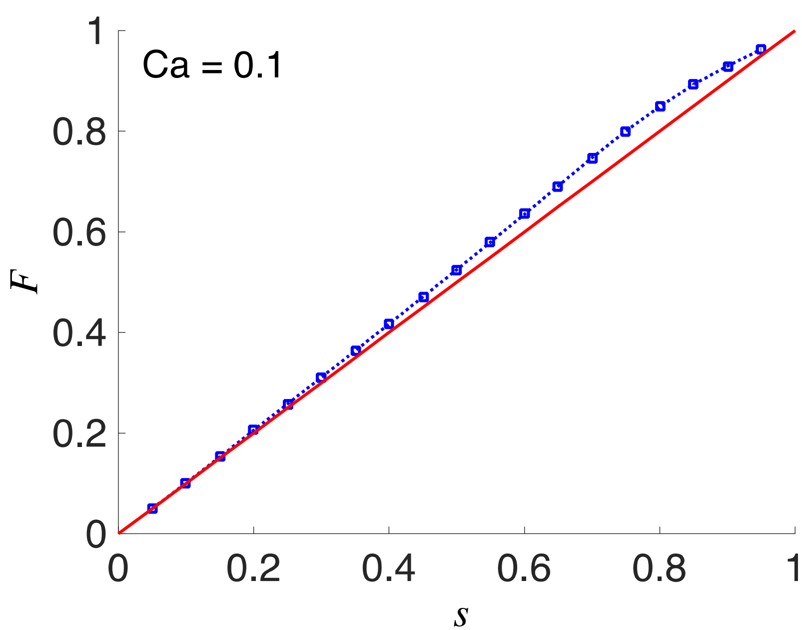
$$F = \frac{\langle |Q_n| \rangle}{\langle |Q| \rangle} = \int_0^\infty dr_{0,ij} \int_0^1 ds_{ij} \int_0^l dx_{b,ij} s_{ij} f(x_{b,ij}, s_{ij}, r_{0,ij}) = \langle s \rangle$$

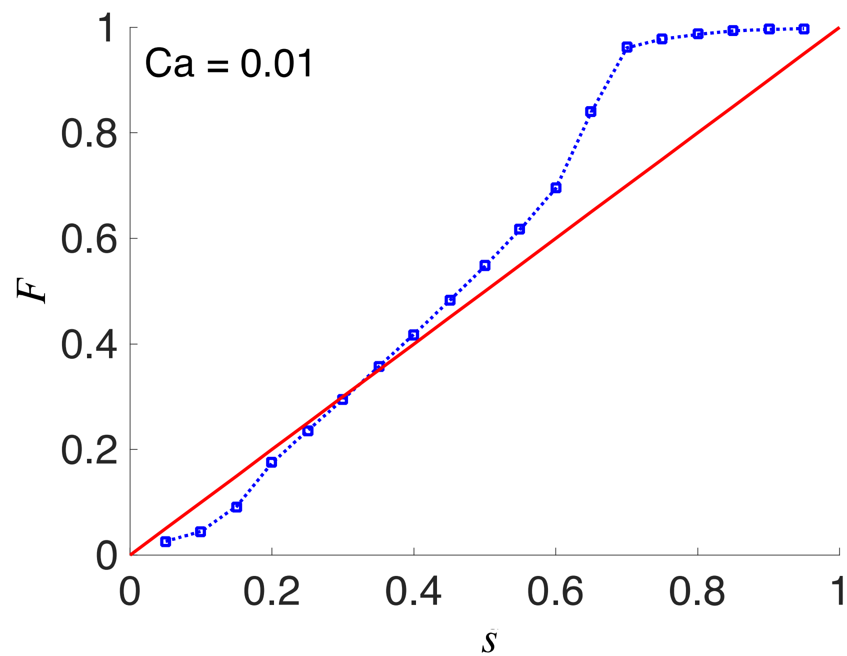




Results

The fractional flow F of the non-wetting fluid is not equal to the volume averaged saturation $s = \langle r_0^2 s \rangle / \langle r_0^2 \rangle$ which is measured





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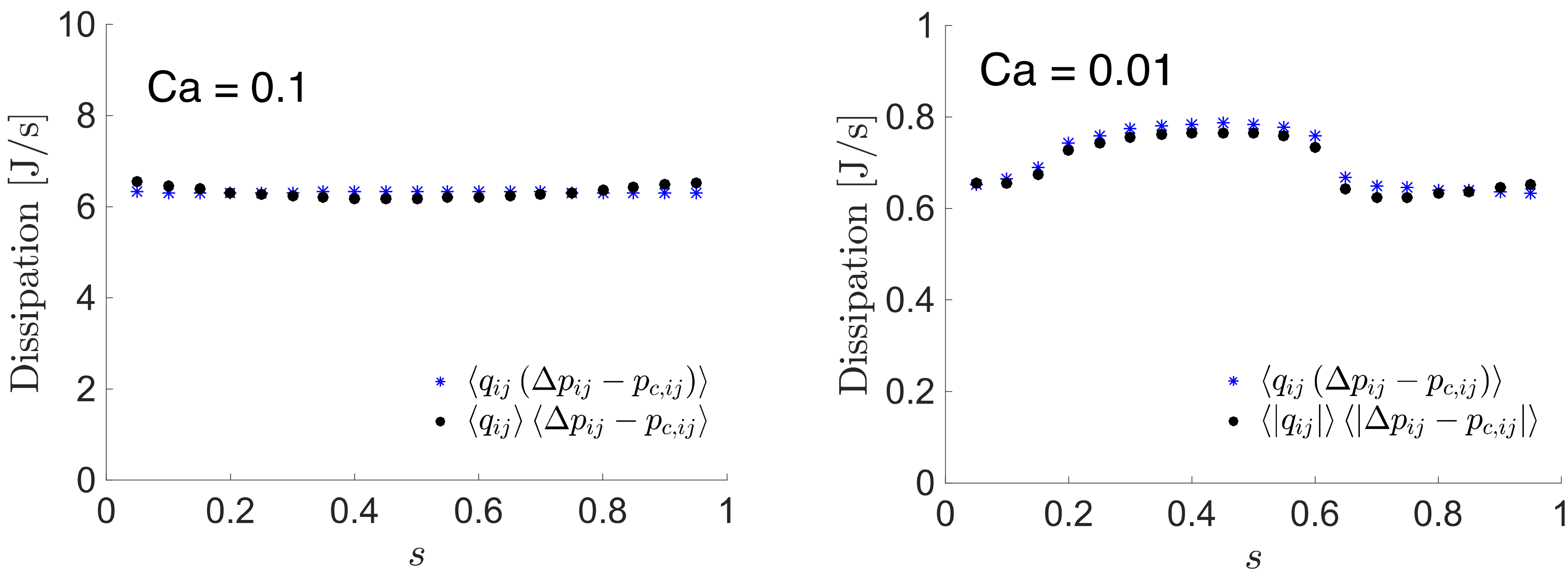
Results

The viscous dissipation is the same, whether we compute it from the distribution function or from overall considerations



Again equal for arbitrary *q* dependence of *f* !!!

PoreLab
NTNU-UiO Porous Media Laboratory

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Senter for fremragende forskning

Co-authors

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- The Faculty of Natural Science and Technology (Vassvik)
- Norwegian Research Council (Sinha)

Thank you for your attention!

Welcome to Trondheim