Thermo-Hydro-Mechanical Coupling in Geomechanics

S. Ali Ghoreishian Amiri

NTNU

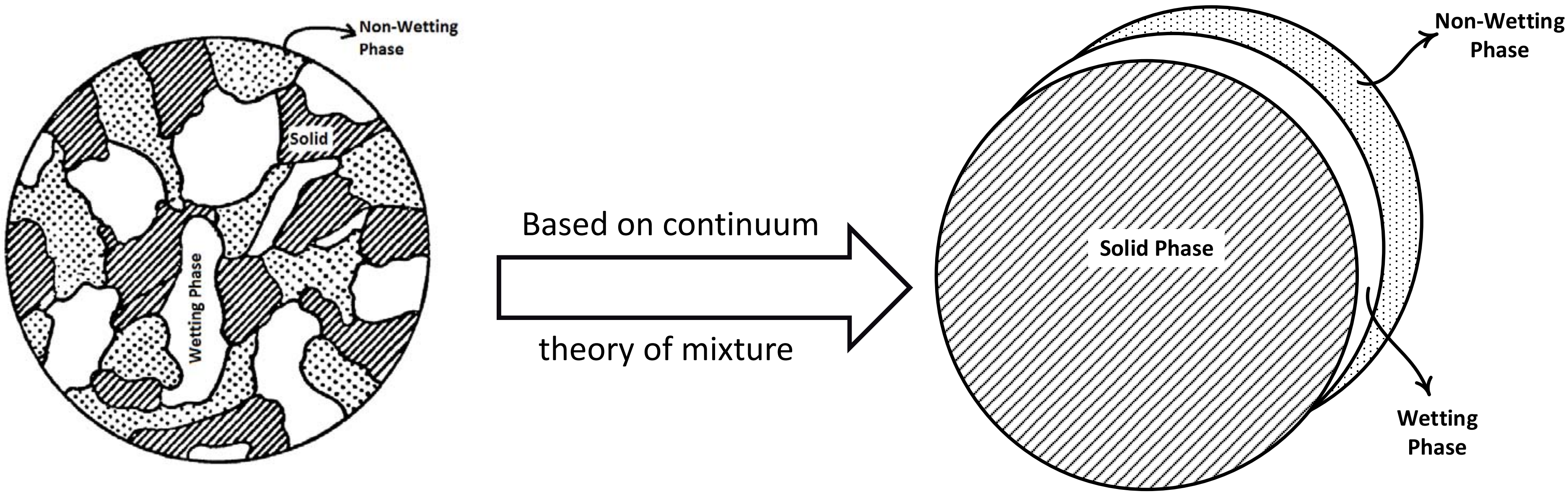
Overview

- **Introduction**
- **Governing equations**
- **Numerical Solution**
- **Verification**

Introduction

- Hydrocarbon Reservoirs
- Radioactive waste repositories
- Geothermal energy utilization
- Partially saturated soils
- Frozen soils

Governing Equations



Basic assumptions:

1- Each point in the domain is simultaneously occupied with all of the phases and based on their volume fraction

2- Local thermal equilibrium between the phases

3- Small strain

Description:

- Solid phase: **Lagrangian**
- Fluid phases: **Eulerian** with respect to the solid phase

Governing Equations

➢ Flow equations

Wetting phase mass balance: $$\frac{D^s[n_w\rho_w]}{Dt}+\nabla.(n_w\rho_w\boldsymbol{w}_w)+n_w\rho_w\nabla.\boldsymbol{v}_s=\dot{M}_w$$

Non-wetting phase mass balance: $$\frac{D^s[n_n\rho_n]}{Dt}+\nabla.(n_n\rho_n\boldsymbol{w}_n)+n_n\rho_n\nabla.\boldsymbol{v}_s=\dot{M}_n$$

- Darcy's law

Wetting phase : $$n_w\boldsymbol{w}_w=\frac{\boldsymbol{K}k_{rw}}{\mu_w}[\rho_w\boldsymbol{g}-\nabla p_w]$$

Non-wetting phase : $$n_w\boldsymbol{w}_w=\frac{\boldsymbol{K}k_{rw}}{\mu_w}[\rho_w\boldsymbol{g}-\nabla p_w]$$

- Soil-water characteristic: $$n_w=F(p_c,T)$$

- Small strain assumption

- Quasi static condition with irrotational deformation field

$$\nabla.\boldsymbol{v}_s=-\frac{D^s\varepsilon_v}{Dt}$$

Governing Equations

➢ Equilibrium equations

Momentum balance of the whole system: $\nabla.\boldsymbol{\sigma} = \rho \boldsymbol{g}$

Effective stress: $\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \frac{1}{n}\boldsymbol{m}\left(n_w p_w + n_n p_n\right)$

Stress-strain relation: $d\boldsymbol{\sigma}' = \boldsymbol{D}_T\left[d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}_0\right]$

Small strain assumption: $\boldsymbol{\varepsilon} = -\frac{1}{2}\left[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right]$

Thermal part of strain: $d\boldsymbol{\varepsilon}_0 = -(\frac{\beta_s}{3}dT)\boldsymbol{I}$

Governing Equations

➢ Heat transfer equations

Energy balance of the whole system: $$\rho C \frac{D^s T}{Dt} + (n_w \rho_w C_w \boldsymbol{w}_w + n_n \rho_n C_n \boldsymbol{w}_n).\nabla T + \nabla.\boldsymbol{q} = Q$$

where:

$$\rho C = (1-n)\rho_s C_s + n_w \rho_w C_w + n_n \rho_n C_n$$

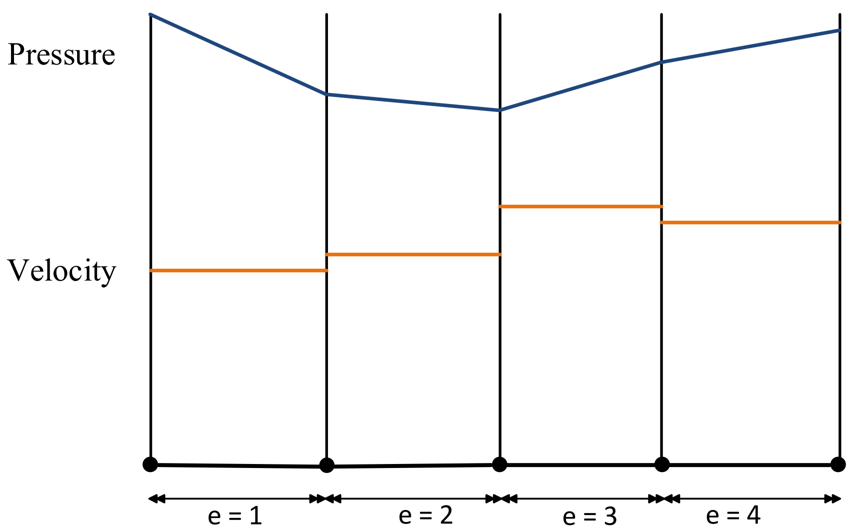
Fourier's law: $$\boldsymbol{q} = -\boldsymbol{\chi}_{eff} \nabla T$$

where:

$$\boldsymbol{\chi}_{eff} = \boldsymbol{\chi}_s^{1-n} . \boldsymbol{\chi}_w^{n_w} . \boldsymbol{\chi}_n^{n_n}$$

Numerical Solution

- **Equilibrium equations: Standard Galerkin FEM**
- **Heat transfer equations: Upwind FEM Schemes**
- **Flow equations: Conservative FEM Scheme**
- **Time discretization: Fully Implicit Scheme**

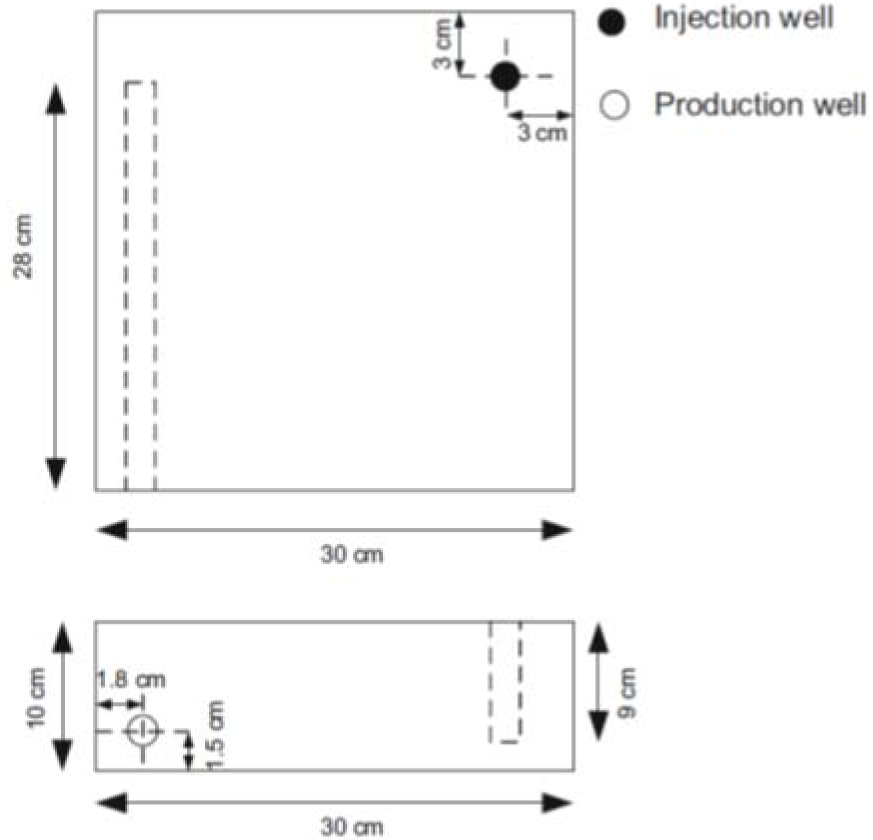


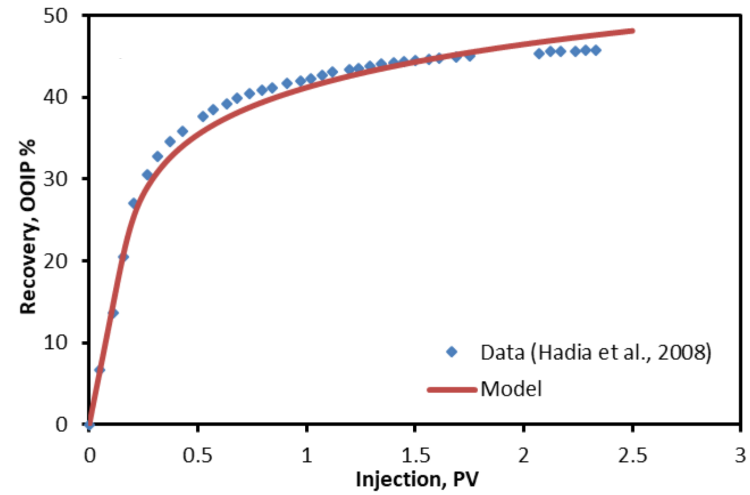
Numerical Solution

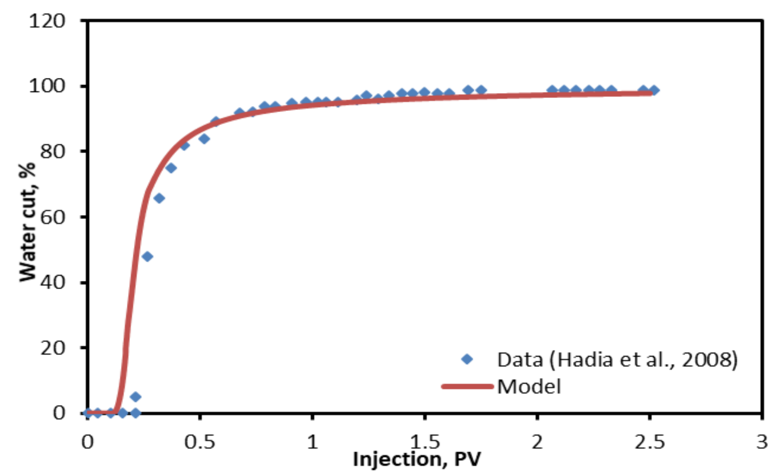
$$\begin{bmatrix} \boldsymbol{P}_{ww}+\Delta t\bar{\boldsymbol{P}}_{ww} & \boldsymbol{C}_{wn} & \boldsymbol{C}_{wu} & \boldsymbol{C}_{wT} \\ C_{nw} & \boldsymbol{P}_{nn}+\Delta t\bar{\boldsymbol{P}}_{nn} & \boldsymbol{C}_{nu} & \boldsymbol{C}_{nT} \\ \boldsymbol{C}_{uw} & \boldsymbol{C}_{un} & \boldsymbol{K}_{uu} & \boldsymbol{C}_{uT} \\ 0 & 0 & 0 & \boldsymbol{T}_{TT}+\Delta t\bar{\boldsymbol{T}}_{TT} \end{bmatrix}_{t+\Delta t} \begin{bmatrix} \Delta\hat{\boldsymbol{p}}_w \\ \Delta\hat{\boldsymbol{p}}_n \\ \Delta\hat{\boldsymbol{u}} \\ \Delta\hat{\boldsymbol{T}} \end{bmatrix}_{t+\Delta t}$$

$$= \Delta t \begin{bmatrix} \boldsymbol{f}_w \\ \boldsymbol{f}_n \\ \dot{\boldsymbol{f}}_u \\ \boldsymbol{f}_T \end{bmatrix}_{t+\Delta t} - \Delta t \begin{bmatrix} \bar{\boldsymbol{P}}_{ww} & 0 & 0 & 0 \\ 0 & \bar{\boldsymbol{P}}_{nn} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\boldsymbol{T}}_{TT} \end{bmatrix}_{t+\Delta t} \begin{bmatrix} \hat{\boldsymbol{p}}_w \\ \hat{\boldsymbol{p}}_n \\ \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{T}} \end{bmatrix}_t$$

Verification



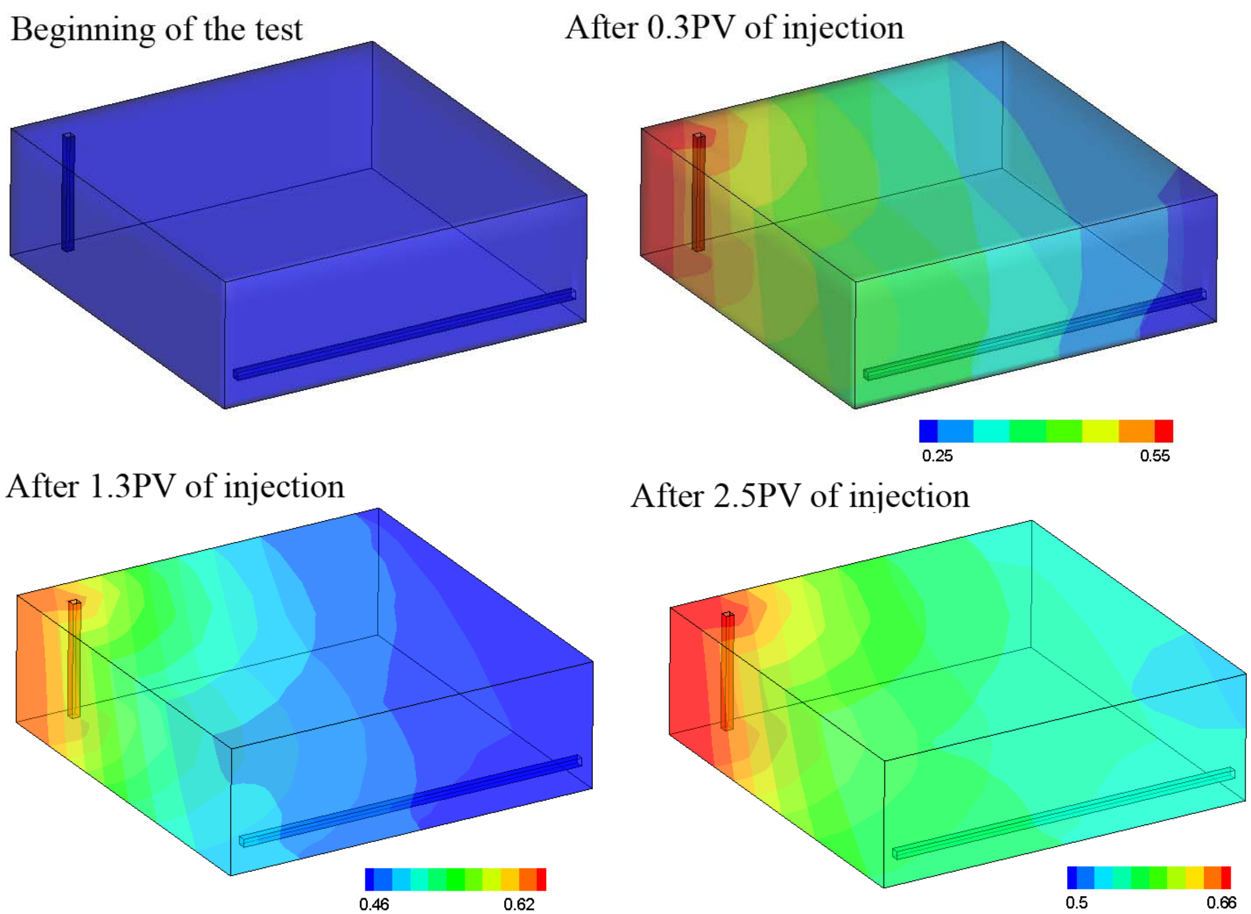




3D water-flooding tests on sandstone core samples (Hadia et al., 2008)

Verification

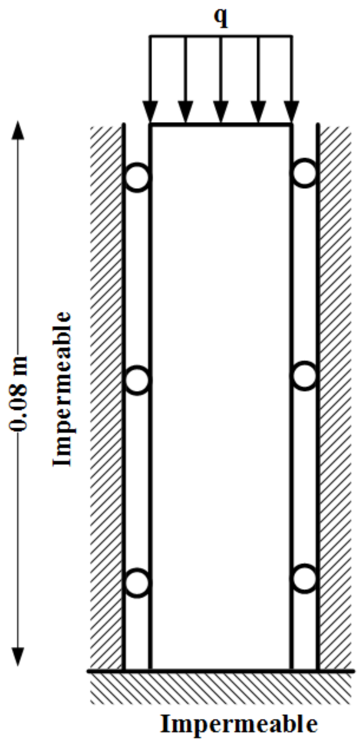
Water saturation:

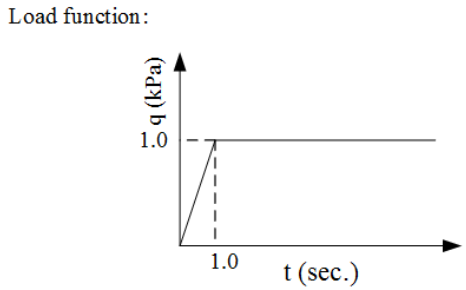


3D water-flooding tests on sandstone core samples (Hadia et al., 2008)

NTNU

Verification





$E = 30$ kPa

$\upsilon = 0.2$

$k = 1.e-5$ $m^2/kPa.sec$

$\rho_w = \rho_n = 0.1$ ton/m^3

$p_{dnw} = 2.$ kPa

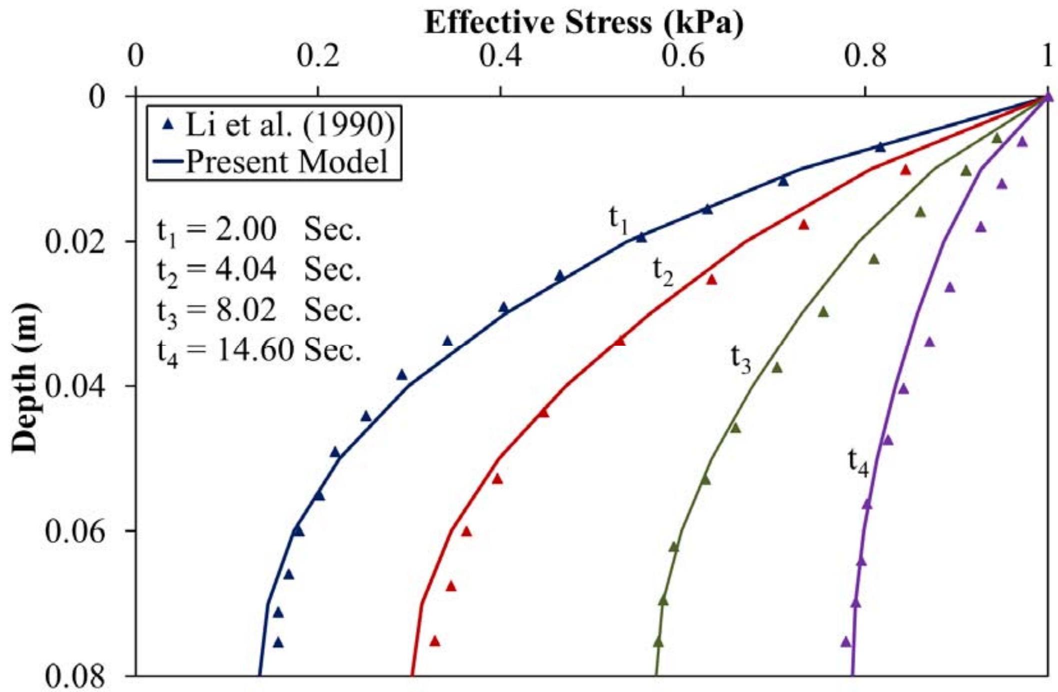
$\lambda = 0.65$

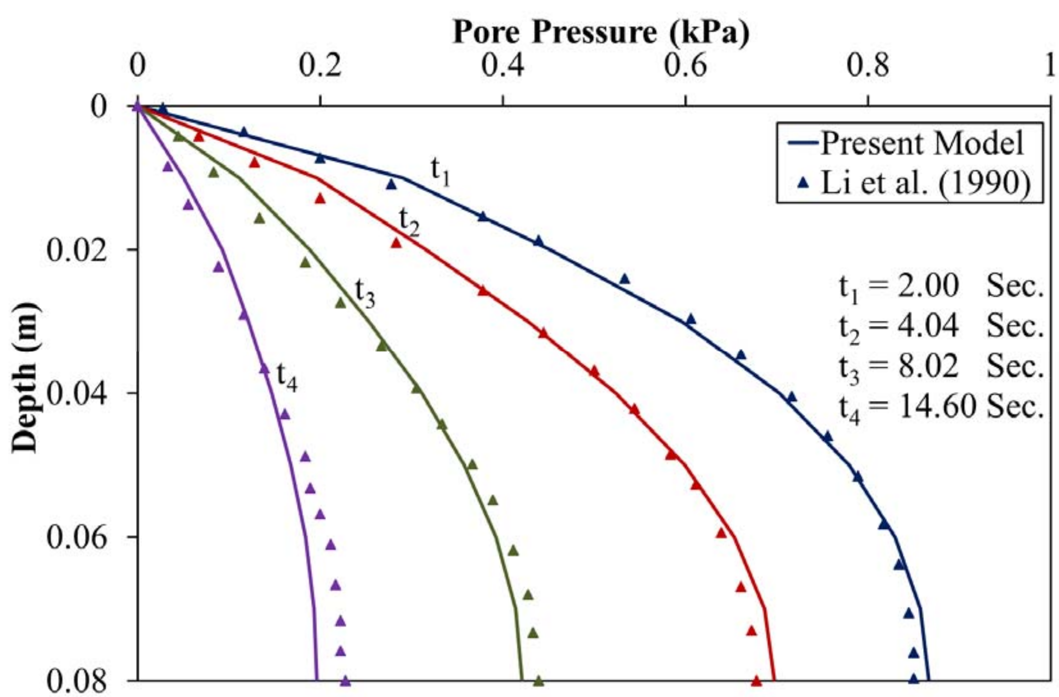
porosity $= 0.322$

initial water saturation $= 0.345$

fluids compressibility $= 0.01$ 1/kPa

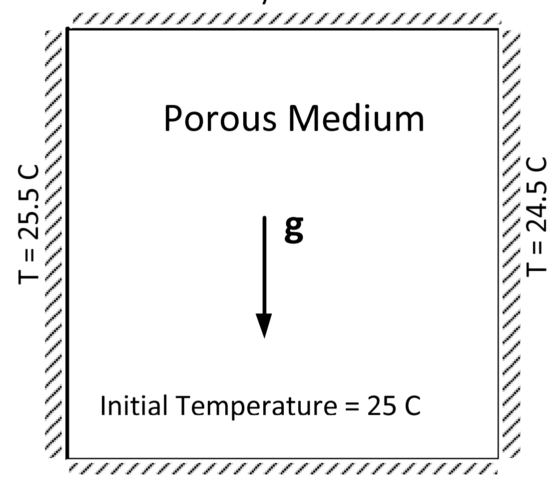
reseidual water volume fraction $= 0.01$

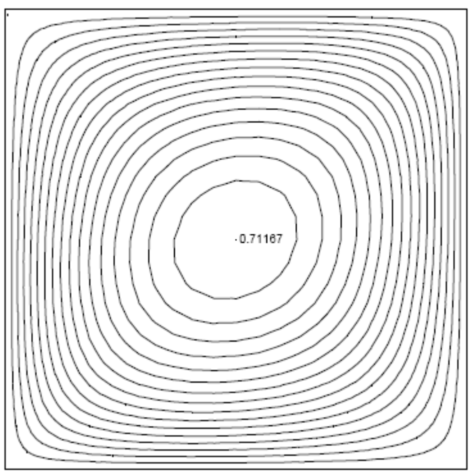




1D-two phase consolidation

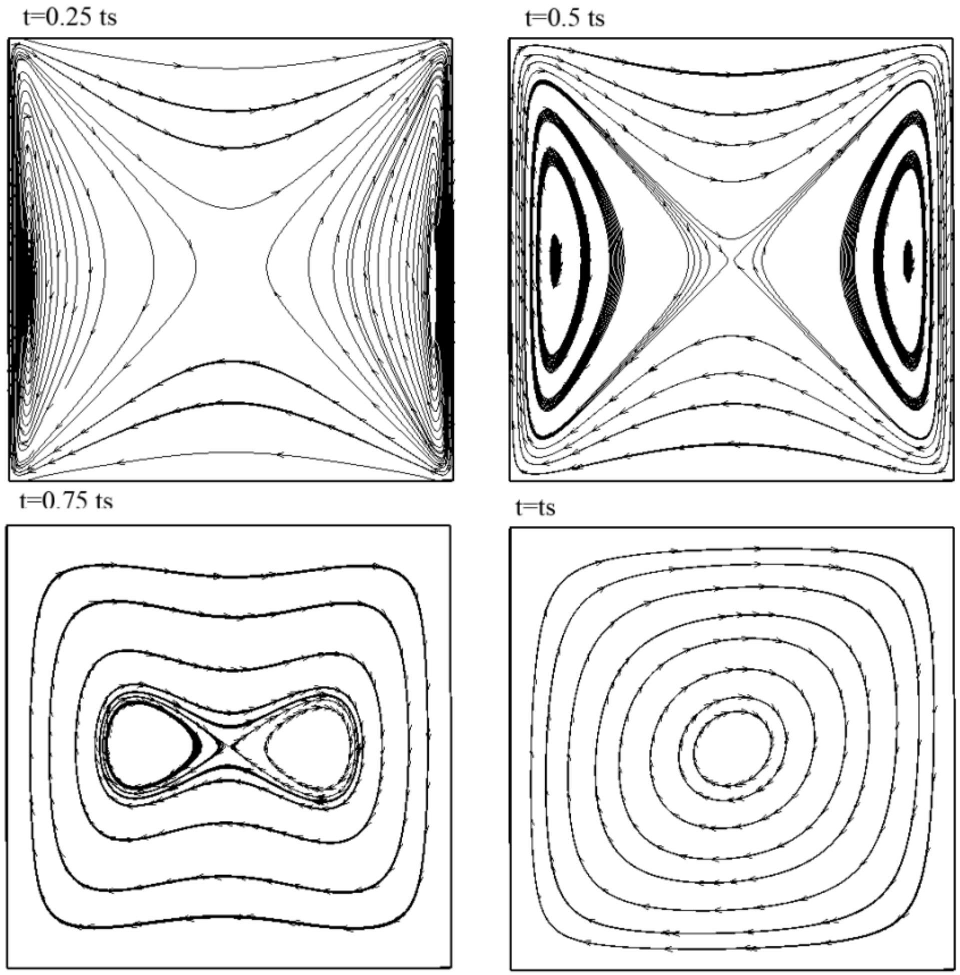
Verification





Massarotti et al., 2001

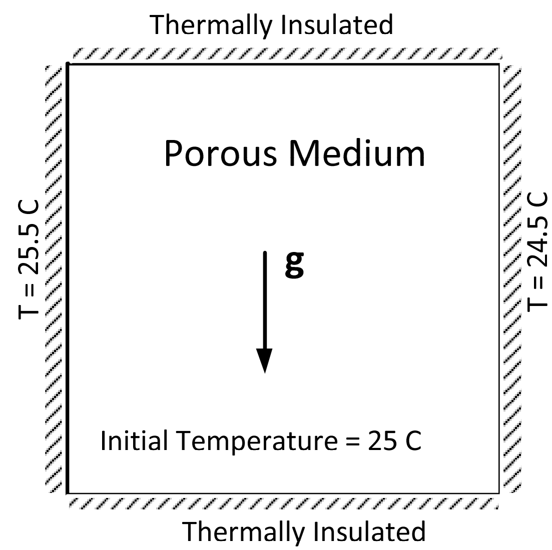
Streamlines:

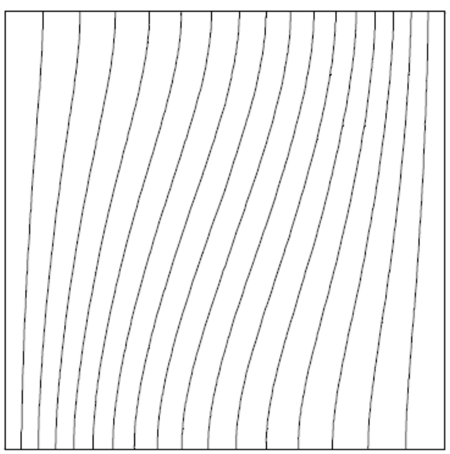


Natural convection due to temperature gradient

NTNU

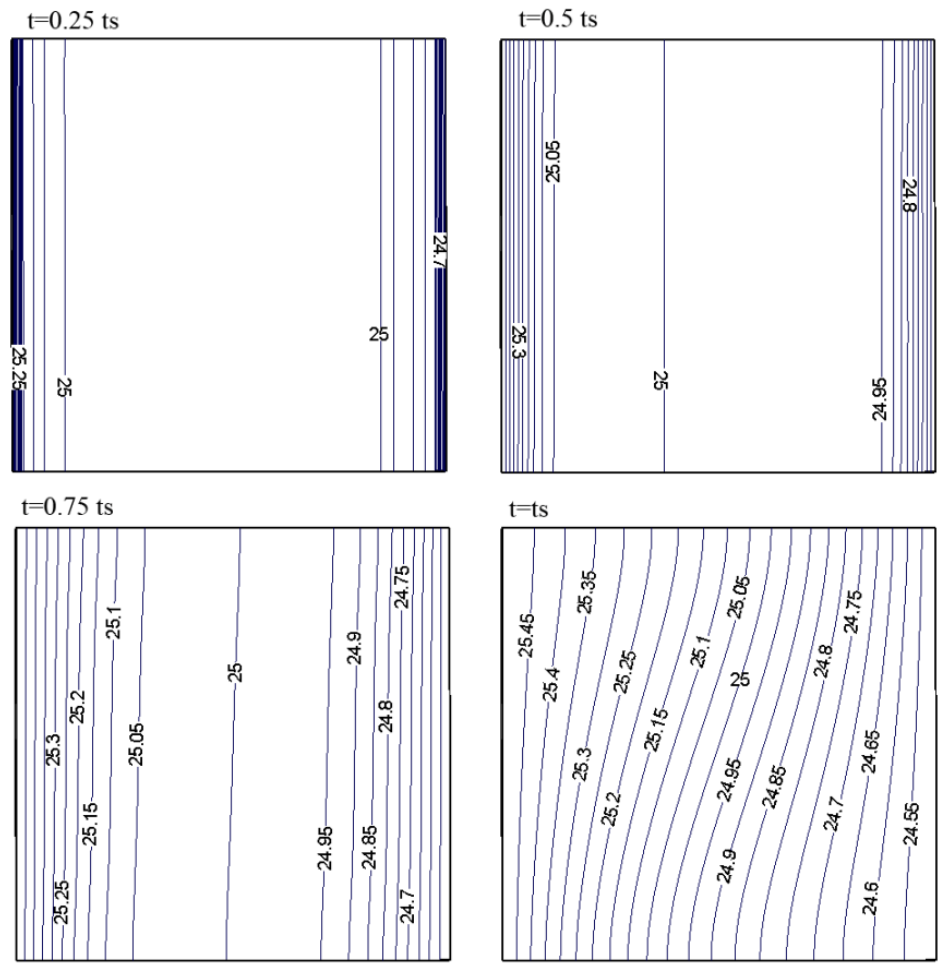
Verification





Massarotti et al., 2001

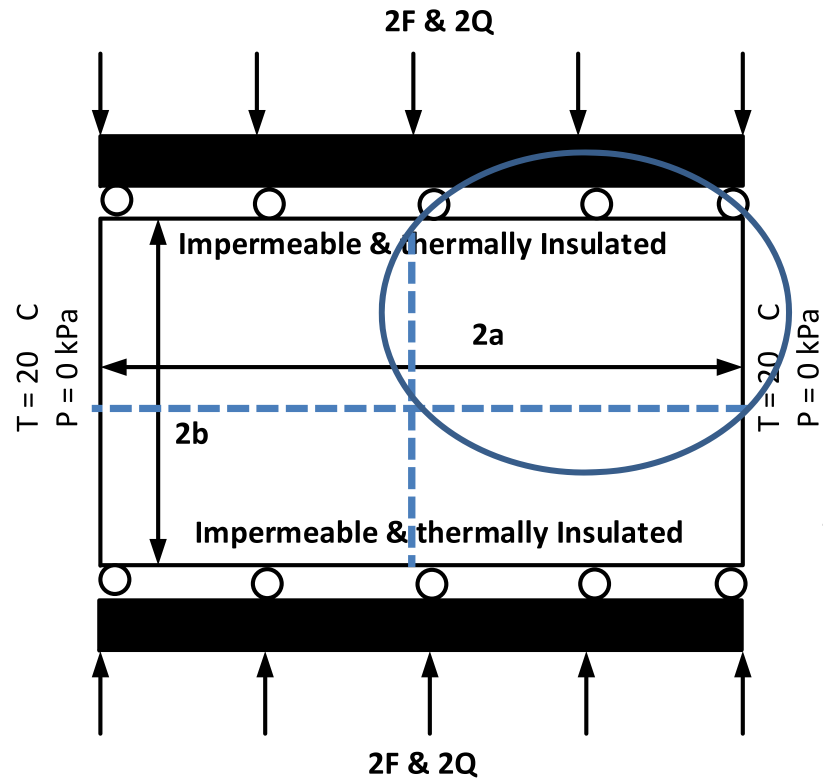
Temperature:



Natural convection due to temperature gradient

NTNU

Verification

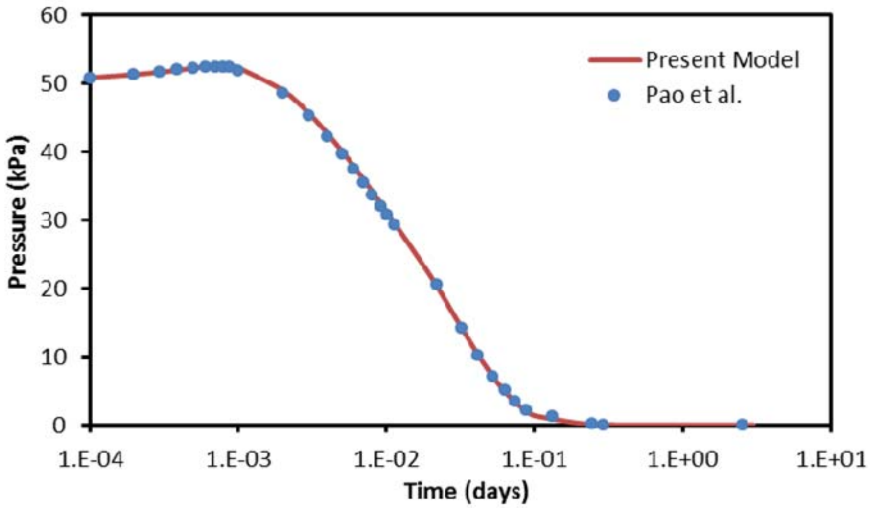


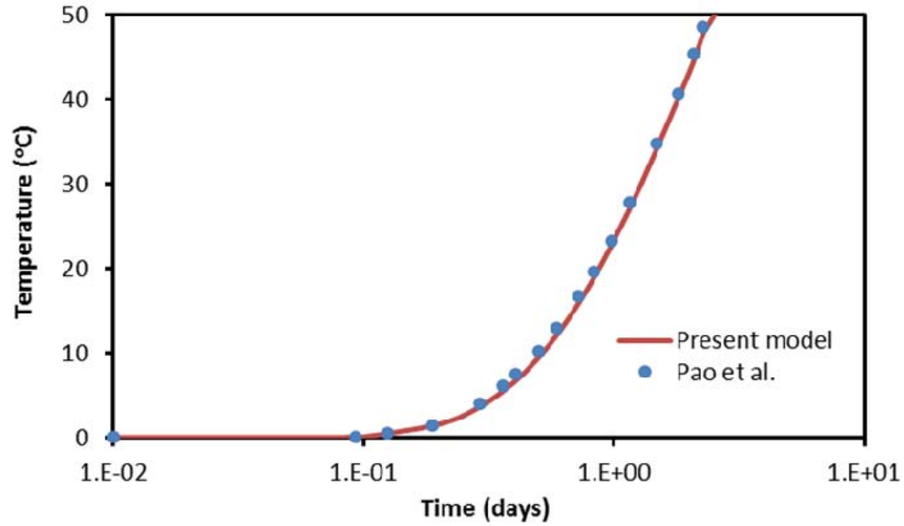
a=b= 1 m
F=1e6 kN
Q=1000 MJ/D

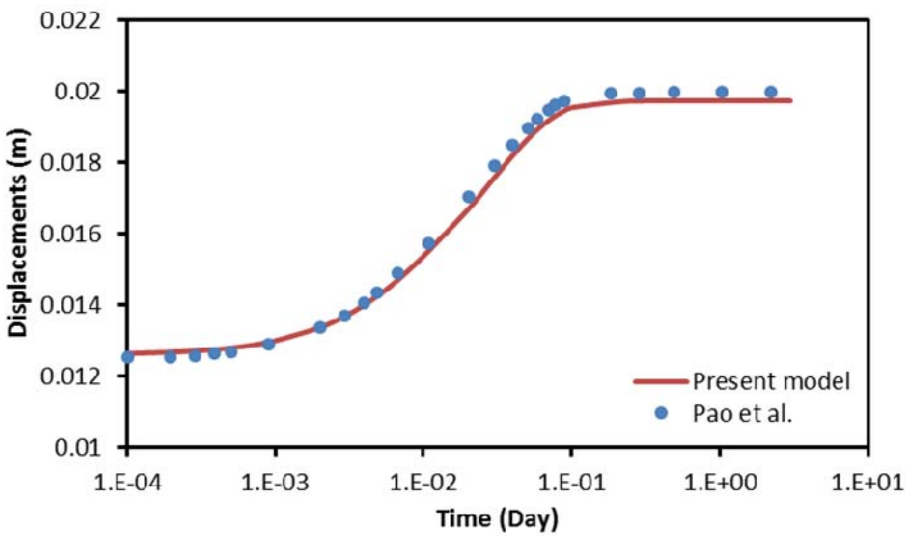
Non-isothermal Mandel problem

NTNU

Verification





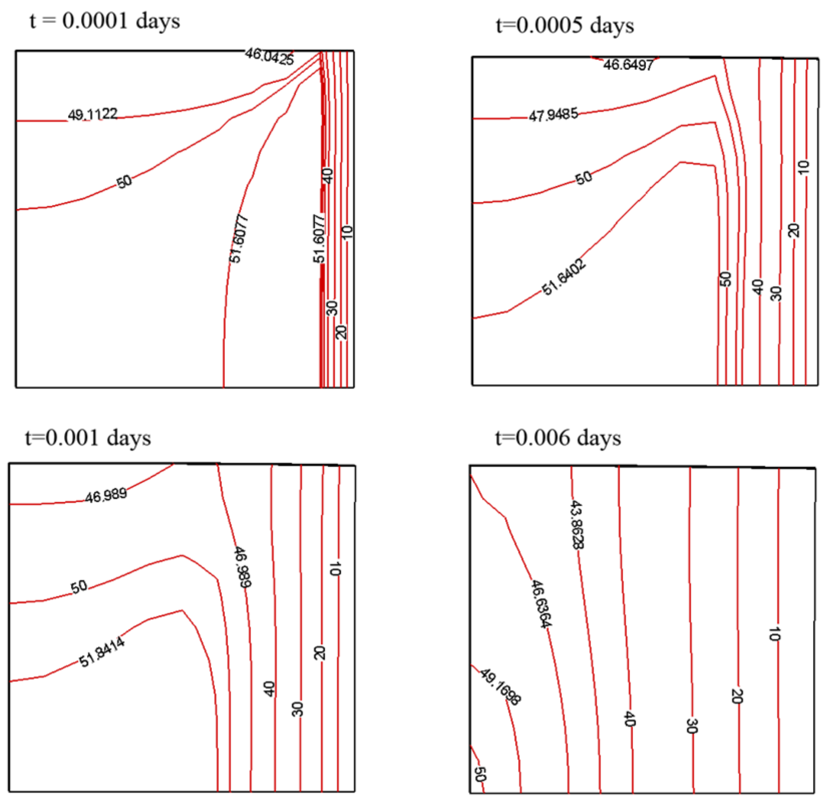


Results at the center of the domain

Non-isothermal Mandel problem

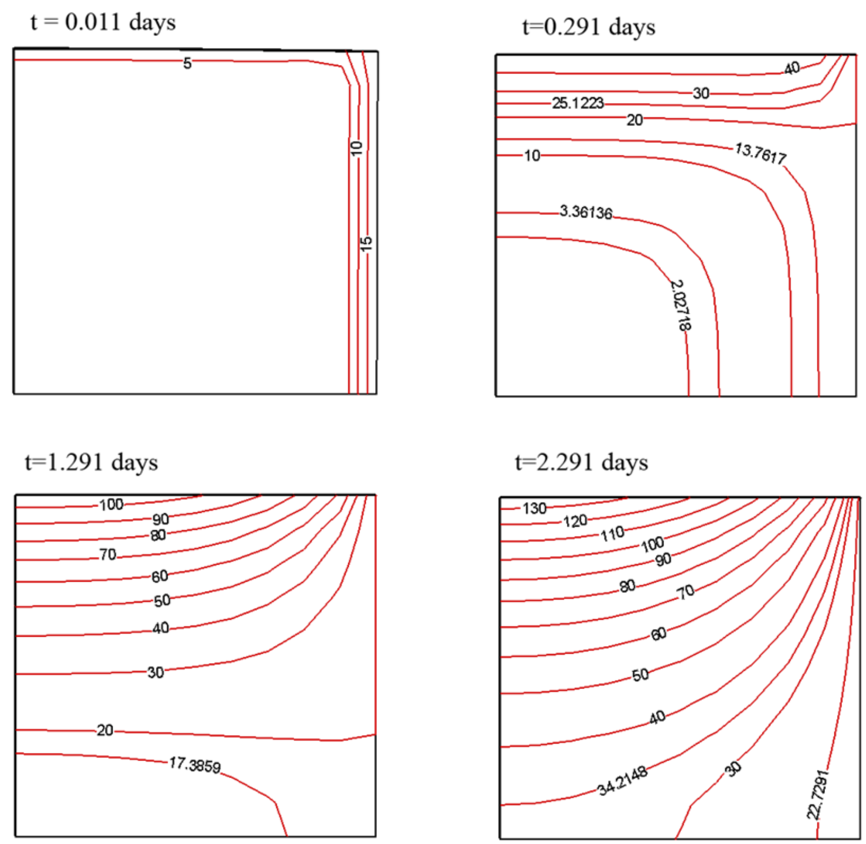
NTNU

Verification



Pressure distribution

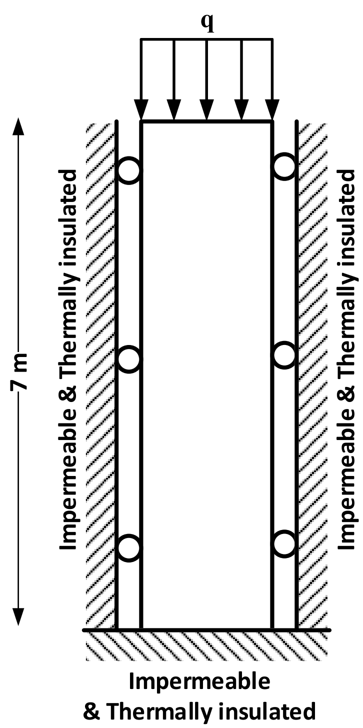
Temperature distribution



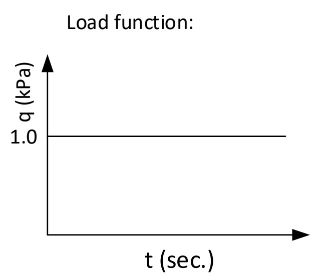
Non-isothermal Mandel problem

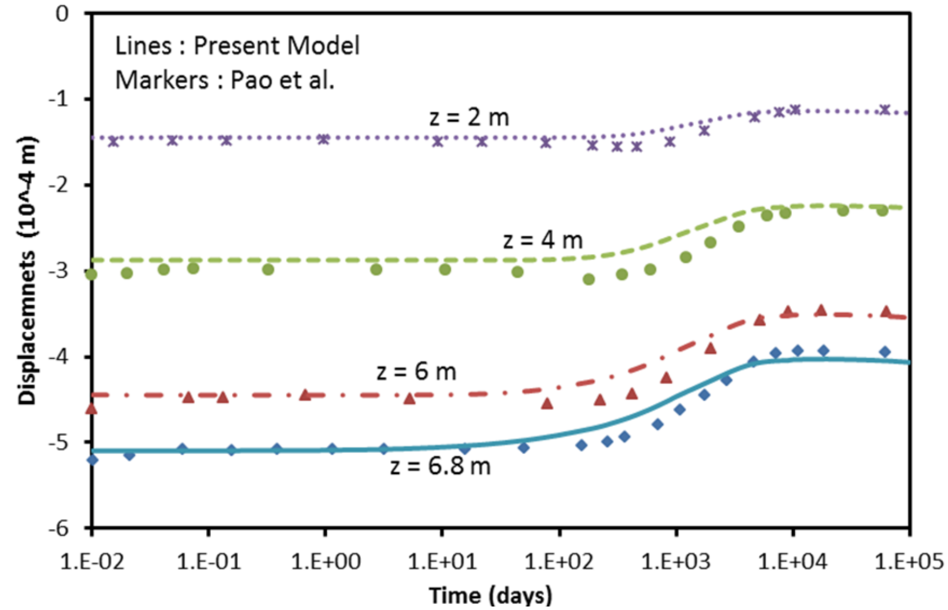
NTNU

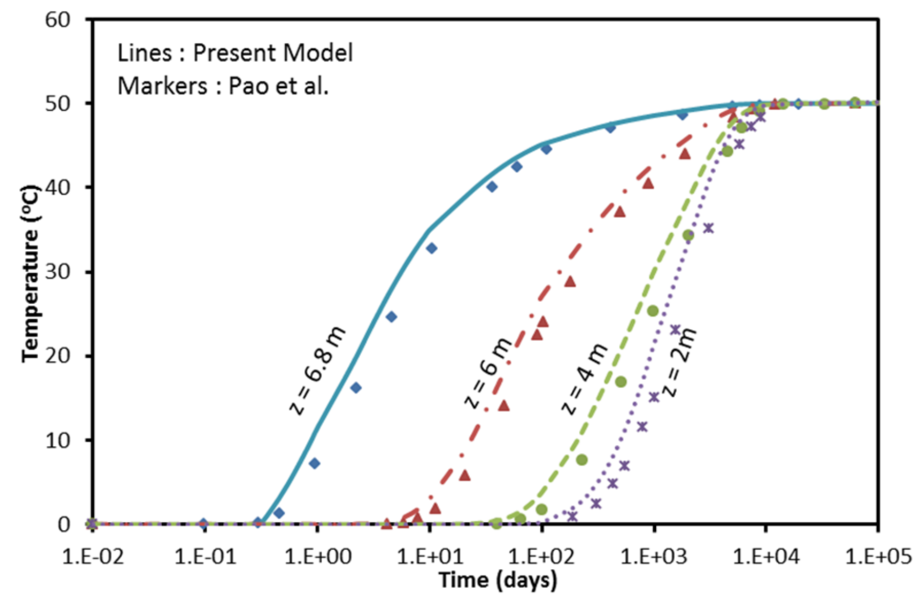
Verification



$p(z=7,t)=0;$
$T(z=7,t)=50^{\circ}C$





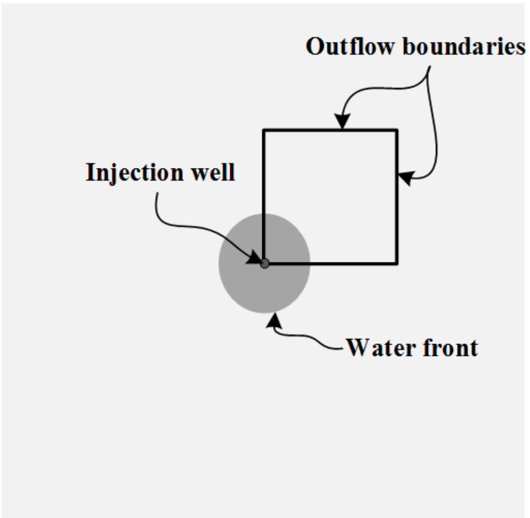


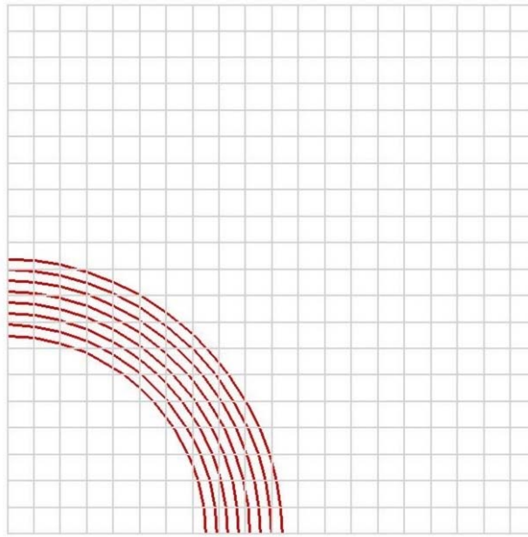
Non-isothermal two phase consolidation

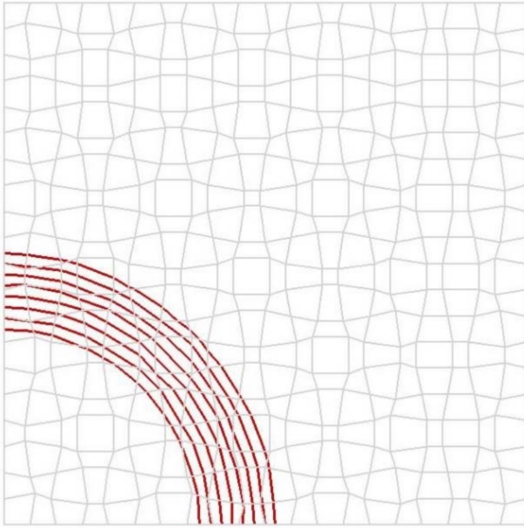
NTNU

Verification

Waterfront:





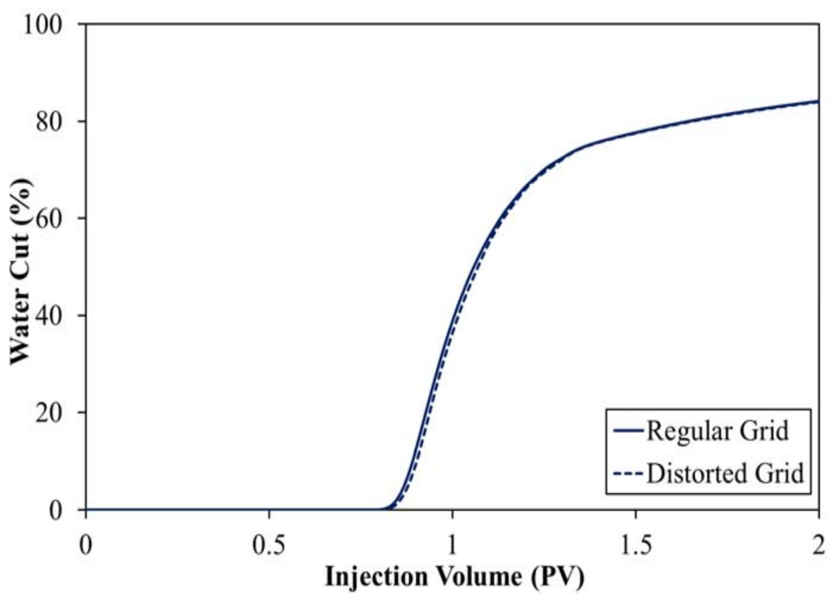


$$k_{rw} = \frac{s^2}{M(1-s^2)+s^2}$$

$$k_{ro} = 1 - k_{rw}$$

where

$$M = \frac{\mu_o}{\mu_w} = 100$$



Radial flow problem: Grid orientation effect

NTNU

Conclusion

- Based on the continuum theory of mixture, a mixture could be modeled as superimposed continua.

- The conservation equations of mass, linear momentum and energy, together with constitutive relations for the pore fluids and solid skeleton constituted the basis of the multi-phase formulation.

- The propose numerical solution preserves local and global conservation of mass and is capable of handling complex geometries and heterogeneities, and dealing with convection dominated problems.

- Different parts of the coupled solution are verified against several benchmark boundary value problems.

Thanks for your attention!