Getting trapped in labyrinths by anomalous diffusion

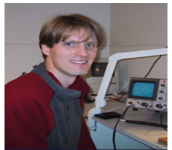
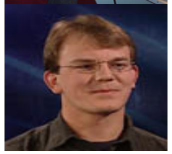
Eirik G. Flekkøy and Knut Jørgen Måløy

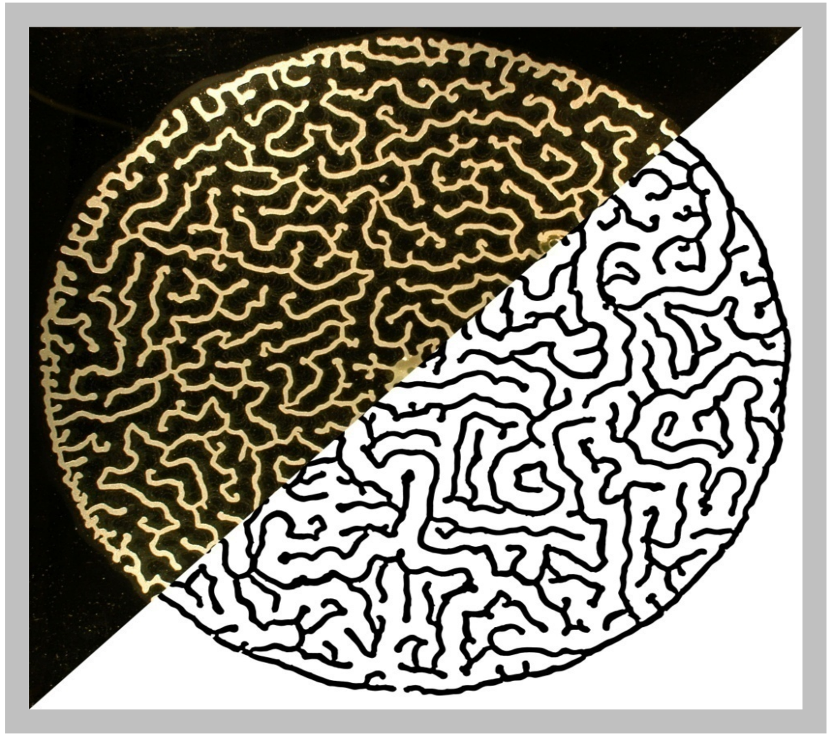
PoreLab
NTNU-UiO Porous Media Laboratory

Senter for fremragende forskning

Labyrints and frictional fluid flow

- Bjørnar Sandnes
- Henning Arendt Knudsen
- Knut Jørgen Måløy
- Jon Alm Eriksen
- Renaud Toussaint

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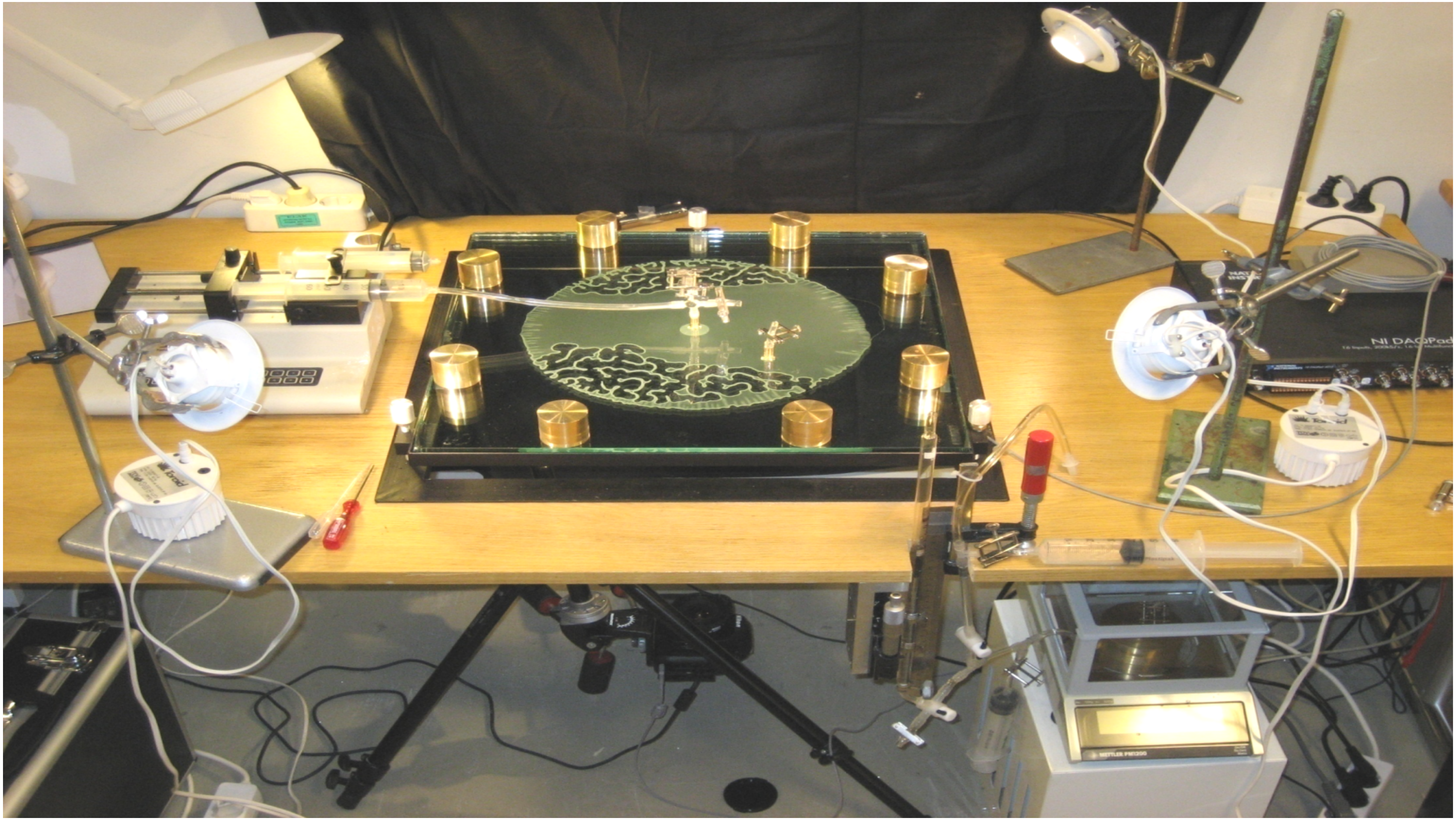
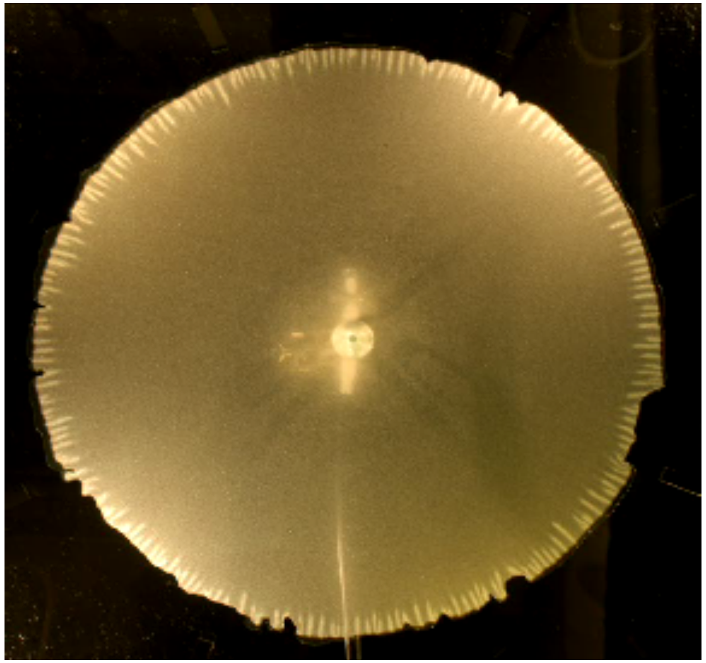

NI DAQPad

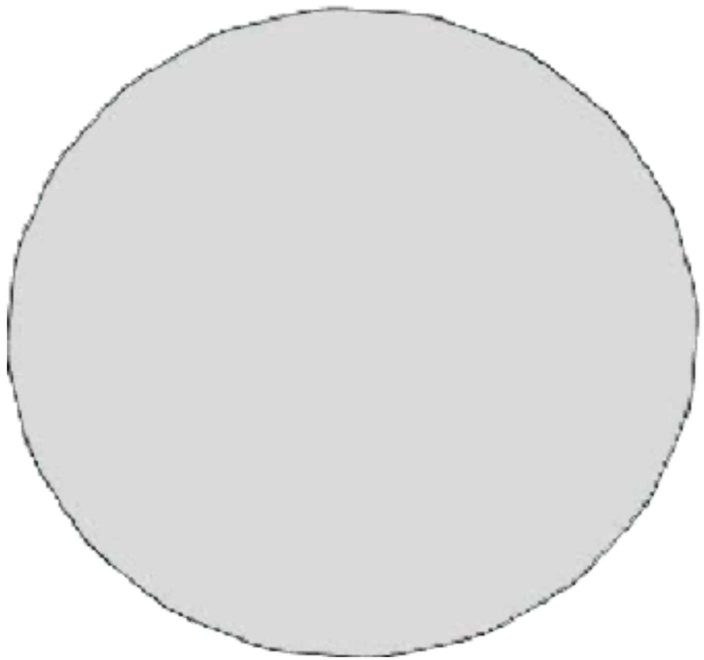
Labyrints and frictional fluid flow

Experiments

simulations



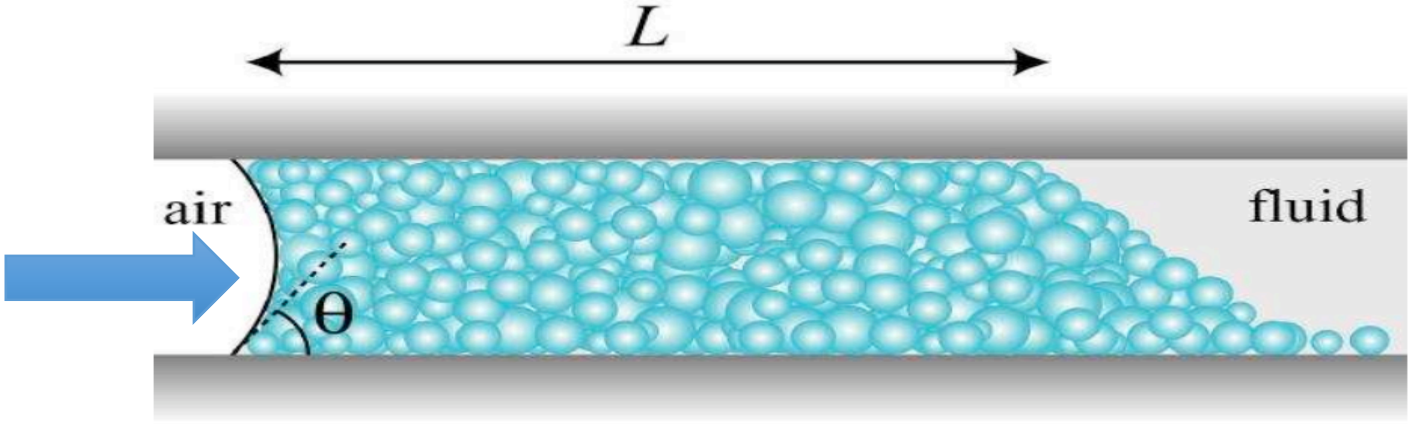
Bjørnar Sandnes



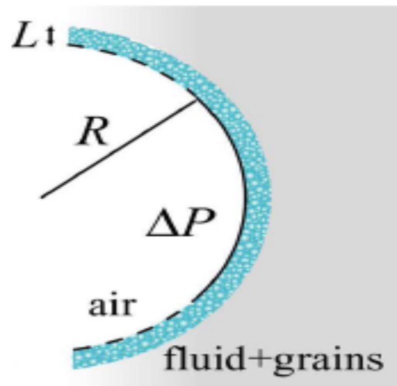
Henning A. Knudsen

Fluid flow with friction and capillarity

Side view:



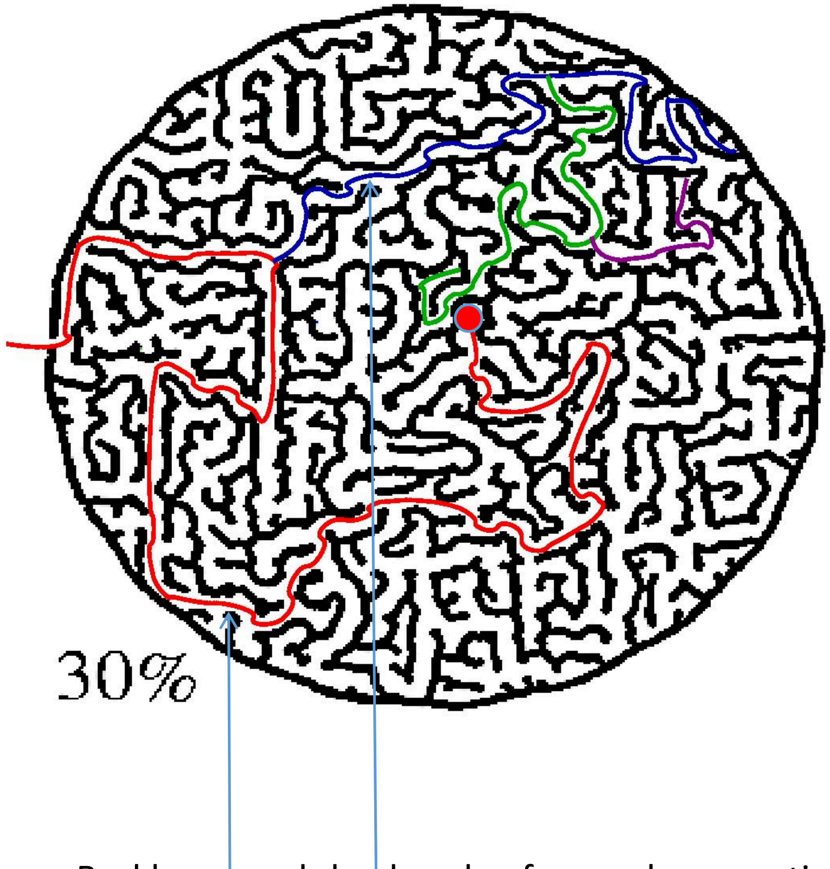
Top view:



Particle-particle interactions transmit stress to the wall

Labyrinth topology

Both displaced (land) and displacing (sea) structures are simply connected.

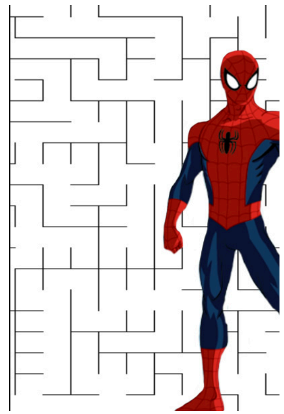


Backbone and dead ends of several generations

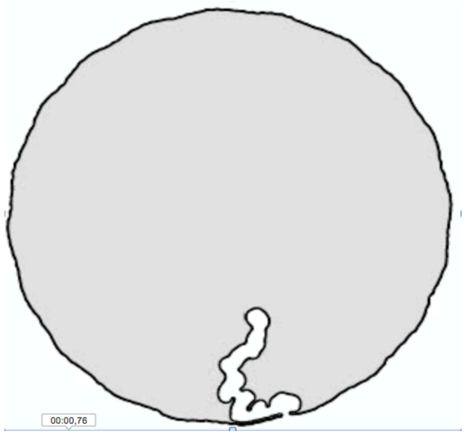
Existing labyrinths

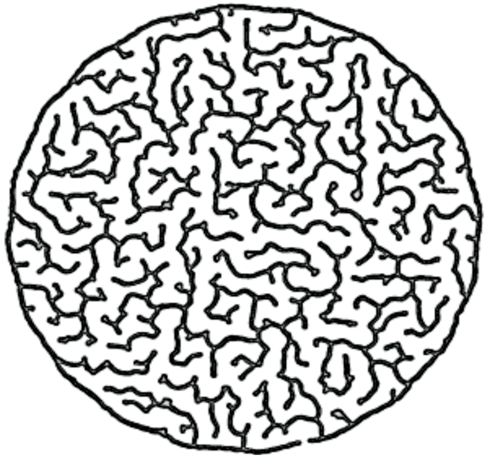
Disconnected element

Existing labyrinths

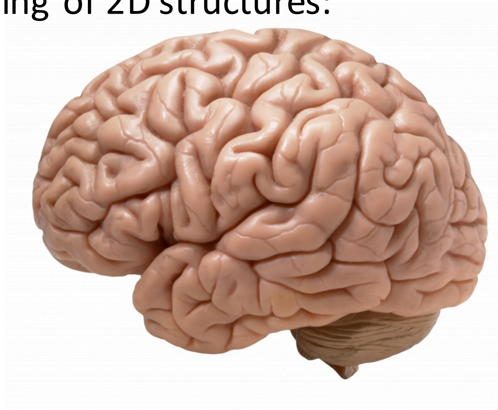


A result of folding of a circle





Folding of 2D structures:

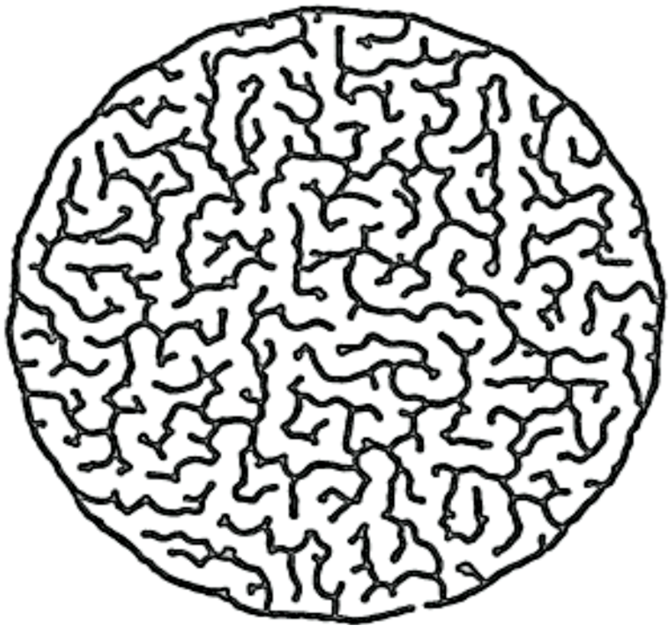


Folding of a 1D structure.

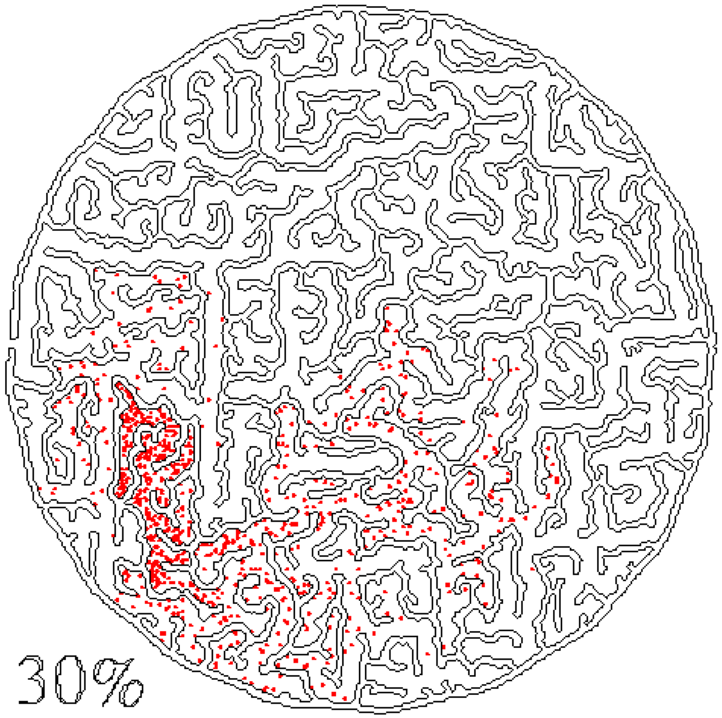
How may the geometry be characterized further ?

Vigeland park (another folded 1D structure)

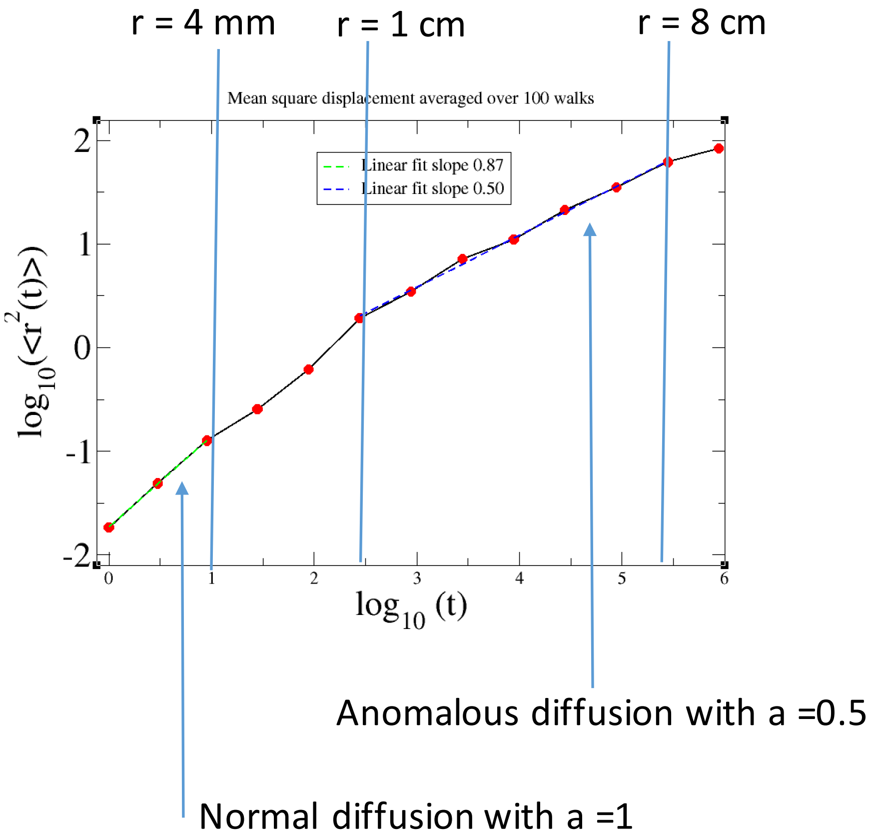
Random walk on labyrinth



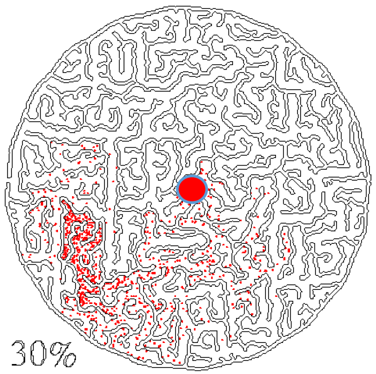
Discretise image,
introduce walker
in center

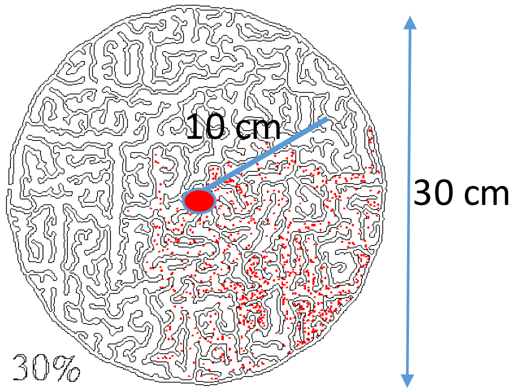


Averaging over random walks started in the center



Simulated labyrinths with finger width 1 cm





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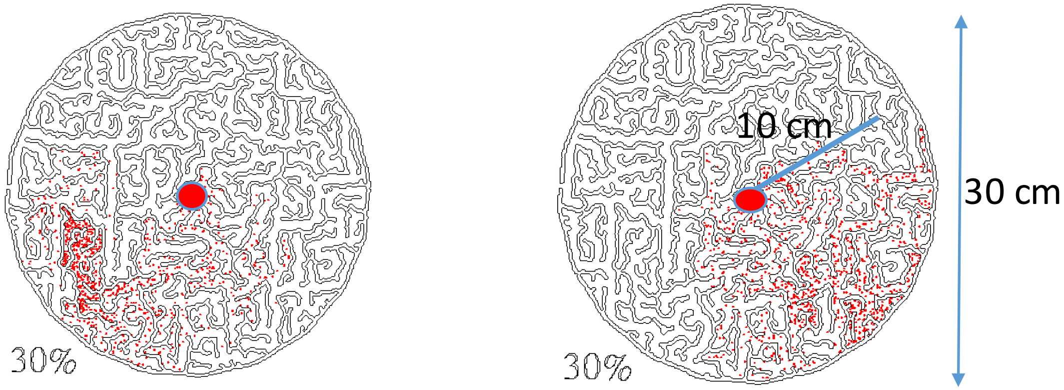
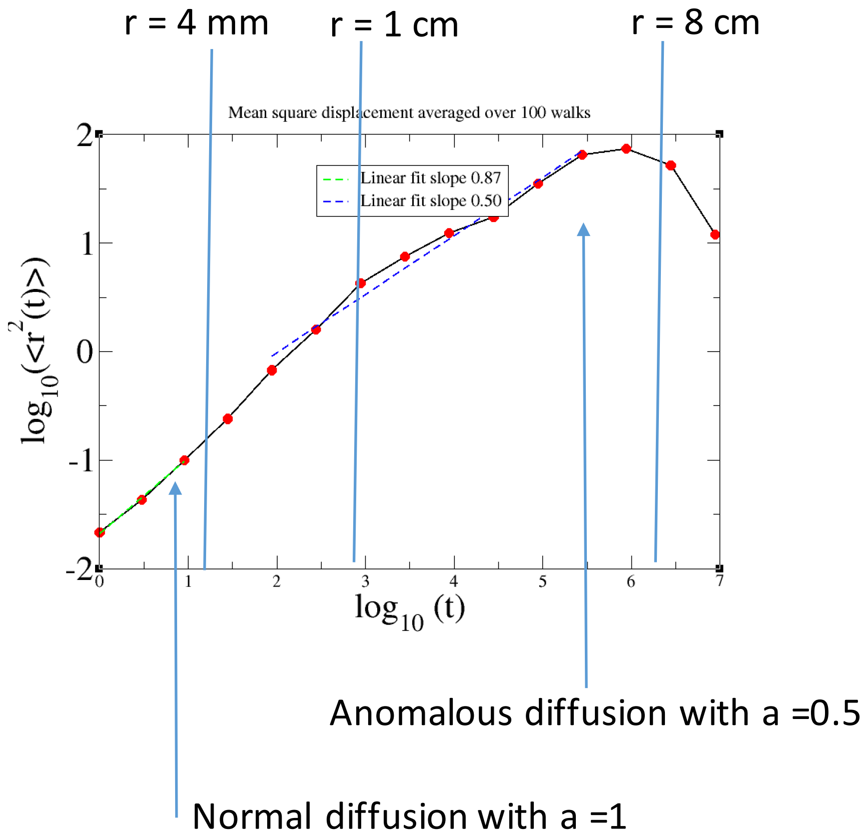
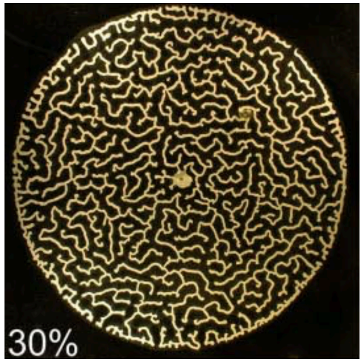
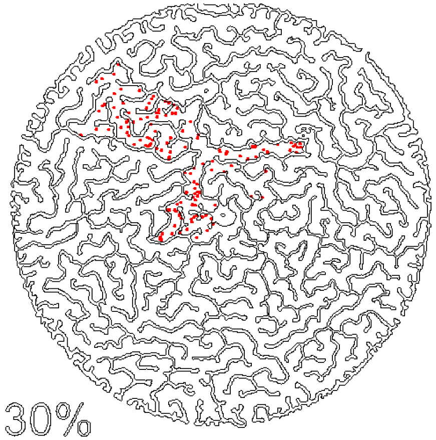
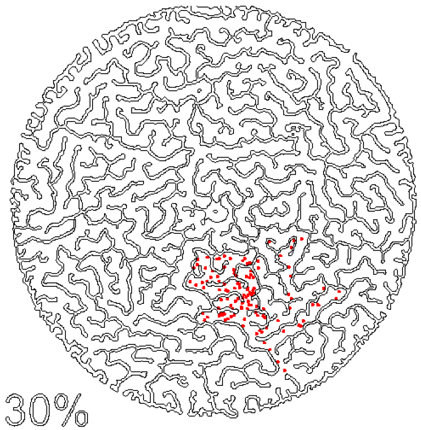


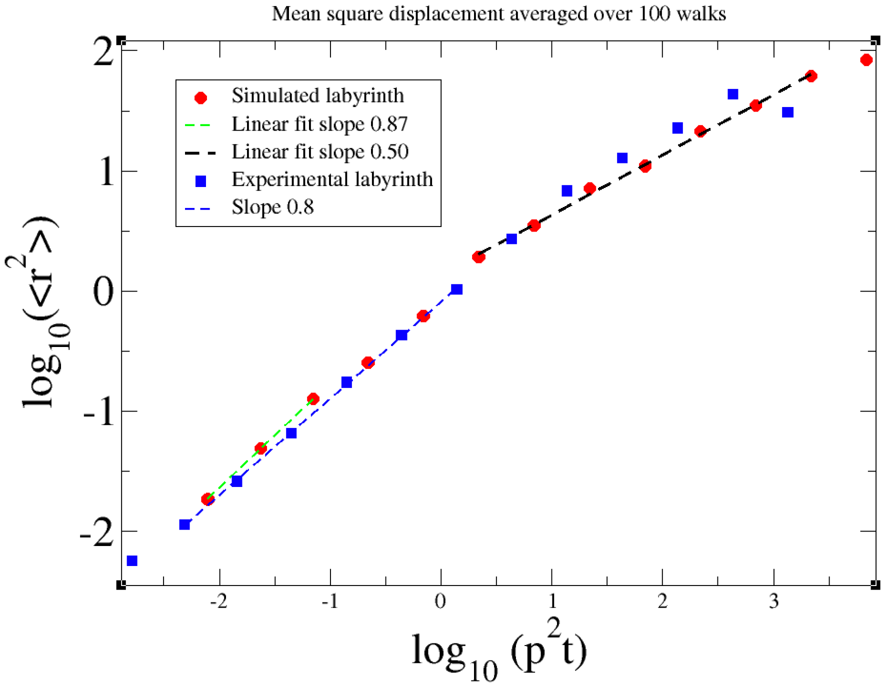


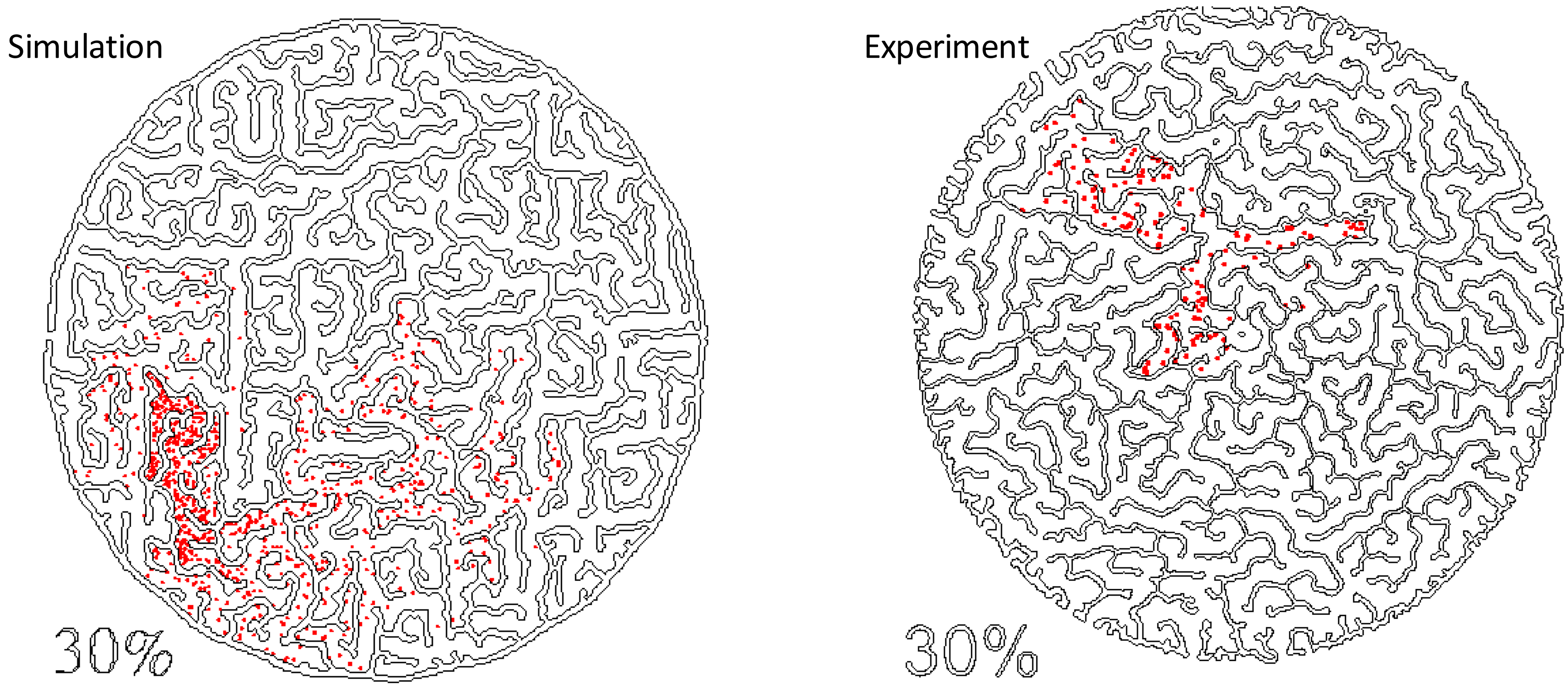
Experimental labyrinths and comparison










Simulation
30%
Experiment
30%

Anomalous diffusion

In general a random walk is described as

$$\langle r^2(t)\rangle = \sum_{n=1}^{N} \langle r_n^2 \rangle = \langle r_n^2 \rangle(t) N(t)$$

Resulting in normal diffusion $\langle r^2(t)\rangle = 2Dt$ if

$$\langle r_n^2 \rangle(t) = a^2$$
$$N(t) = \frac{t}{\tau}$$

Anomalous diffusion happens if this is not the case.

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ANOMALOUS DIFFUSION IN DISORDERED MEDIA: STATISTICAL MECHANISMS, MODELS AND PHYSICAL APPLICATIONS

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Antoine GEORGES**

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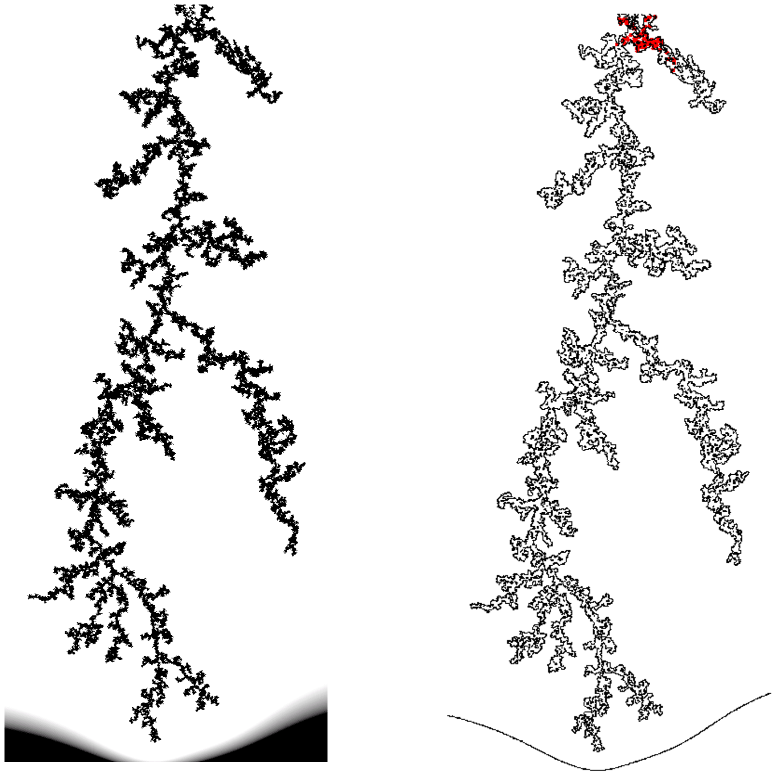
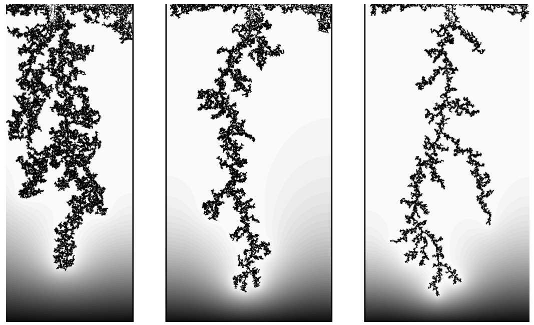
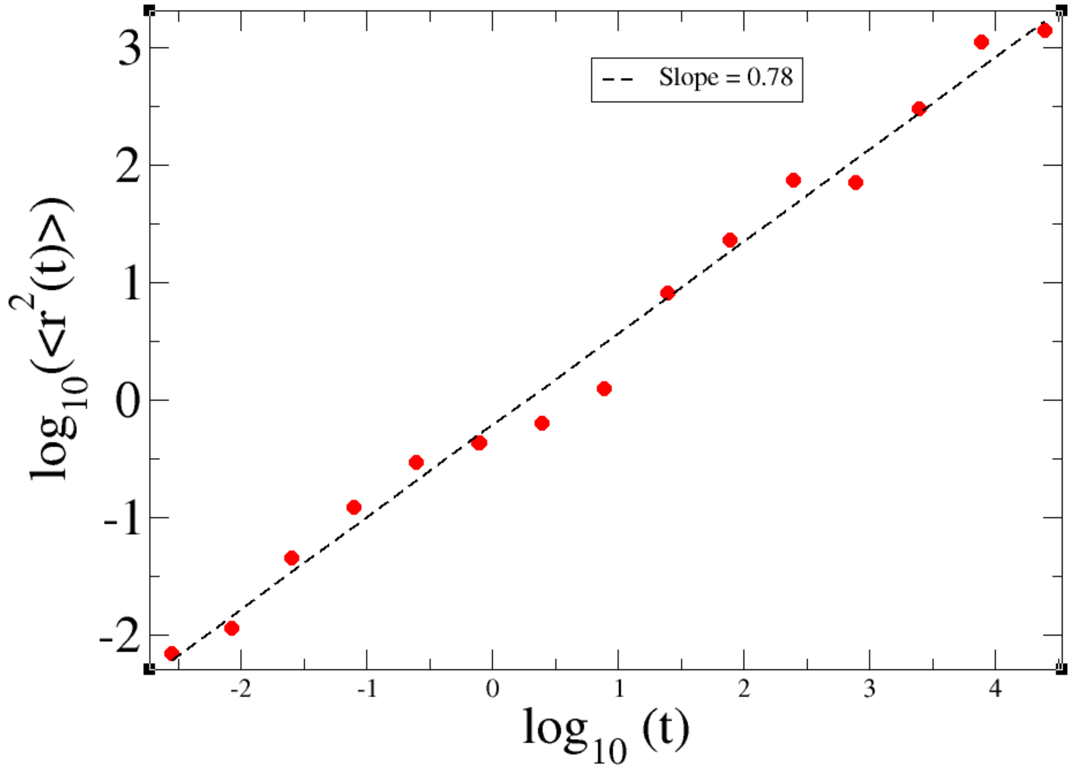
Diffusion on invasion cluster



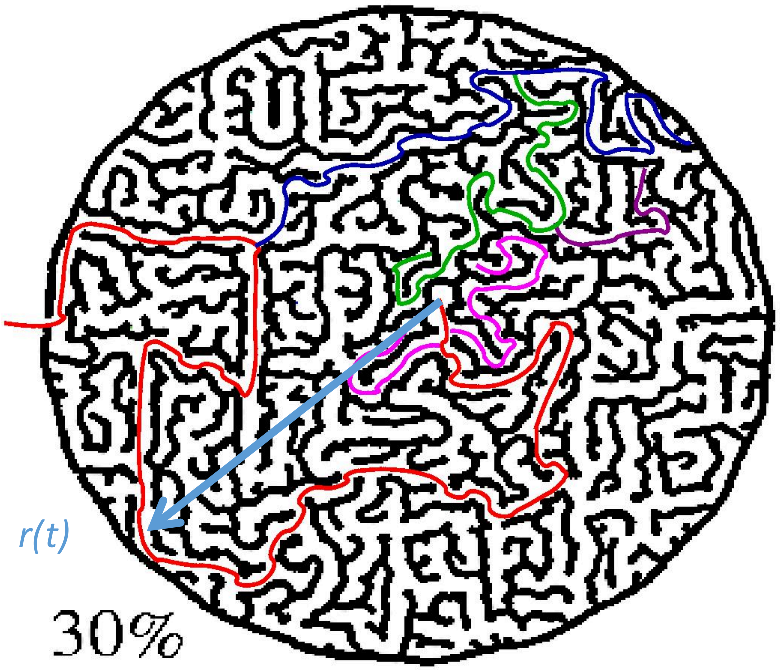


Experiments by K.J. Måløy



The ant in the labyrinth P.G. de Gennes, La Recherche 7 (1976)
Diffusion on percolation clusters.

Theory for subdiffusion due to long waiting times



Length along backbone r_B

PHYSICAL REVIEW E 95, 012139 (2017)

Minimal model for anomalous diffusion

Eirik G. Flekkøy*

Conjecture $\langle r^2(t) \rangle \sim \langle r_B^2(t) \rangle$

$$\langle r_B^2(t) \rangle = \sum_{n=1}^{N} \langle r_n^2 \rangle = \langle r_n^2 \rangle N$$

The waiting times τ to return to the backbone is distributed as

$$P(\tau) \propto \frac{1}{\tau^{3/2}}$$

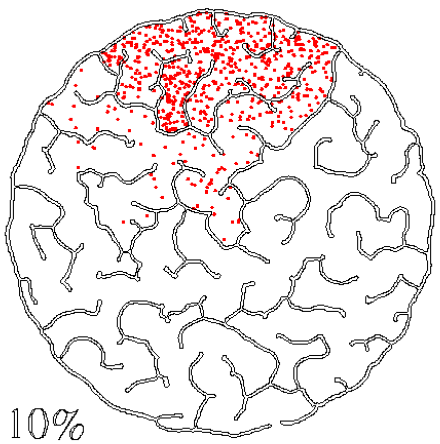
$$N \propto t^{1/2}$$

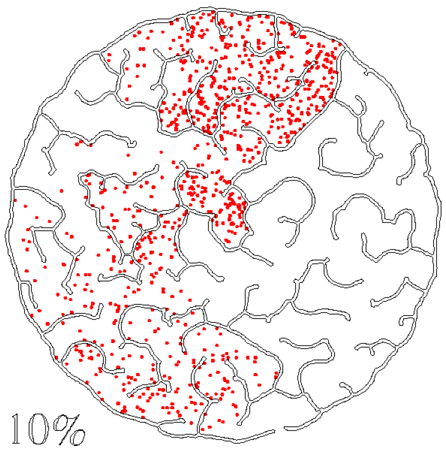
$$\langle r^2(t) \rangle \sim t^{1/2}$$

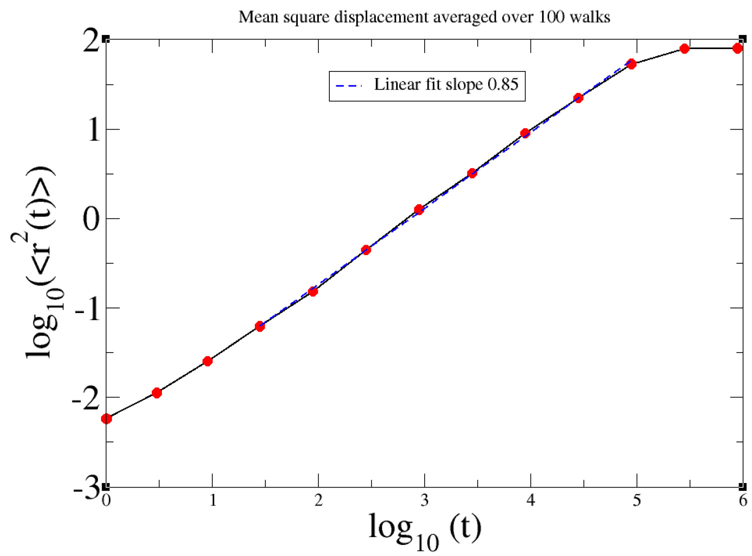
E W Montroll and H Scher, J Stat Phys 9 (1973) 101

Diffusion on combs J Machta, J Phys A 18 (1985)

Lower packing fraction

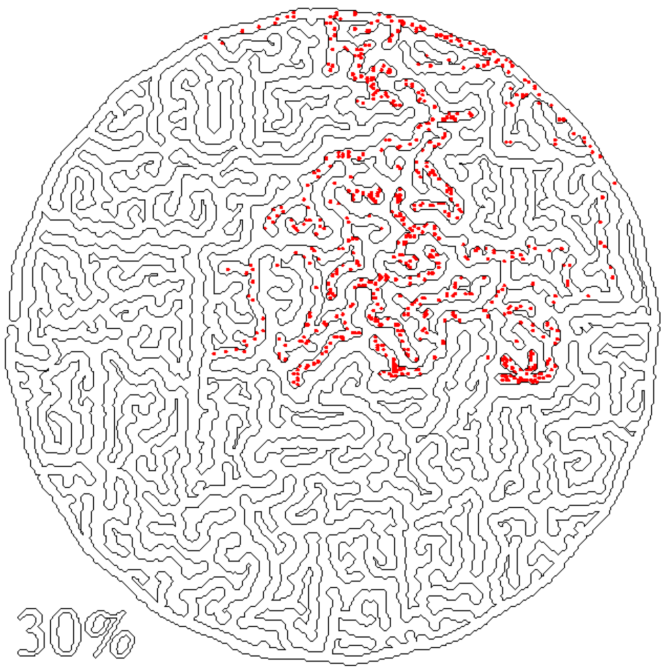


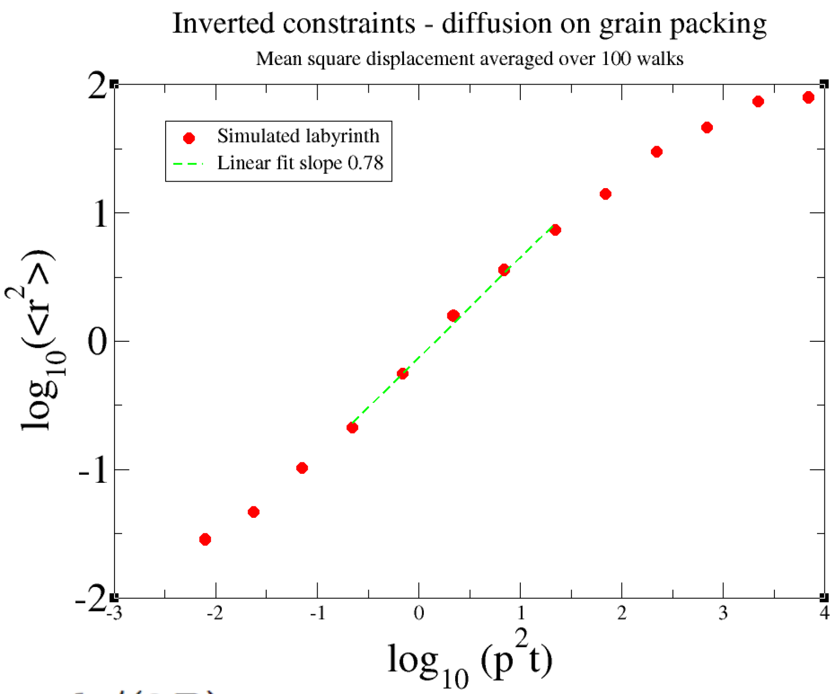




No longer $\tau \ll t_c = l_b/(2D)$

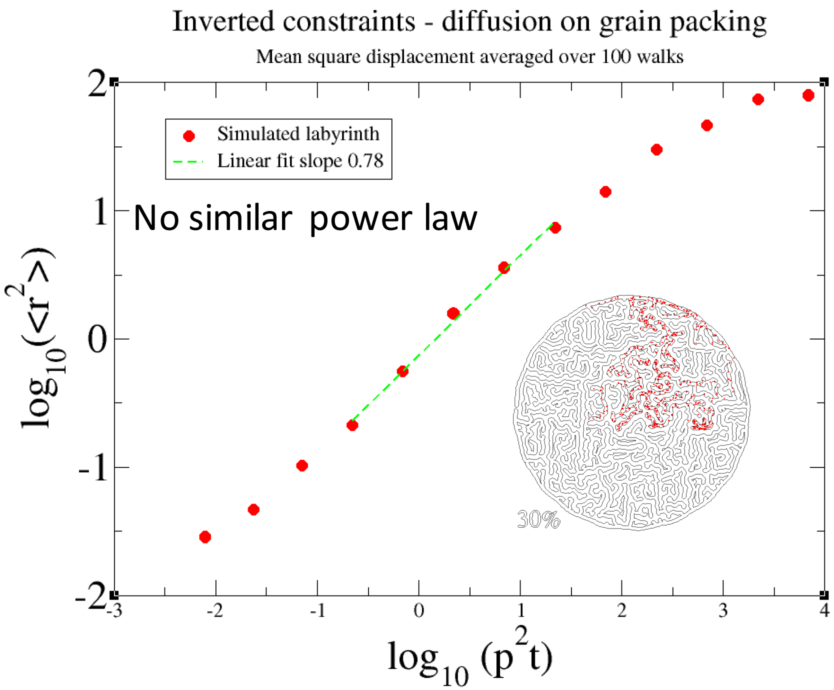
Diffusion on land

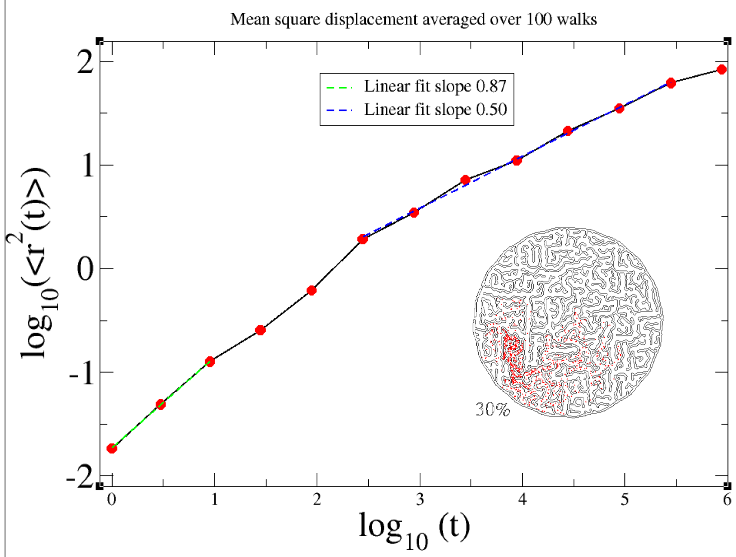




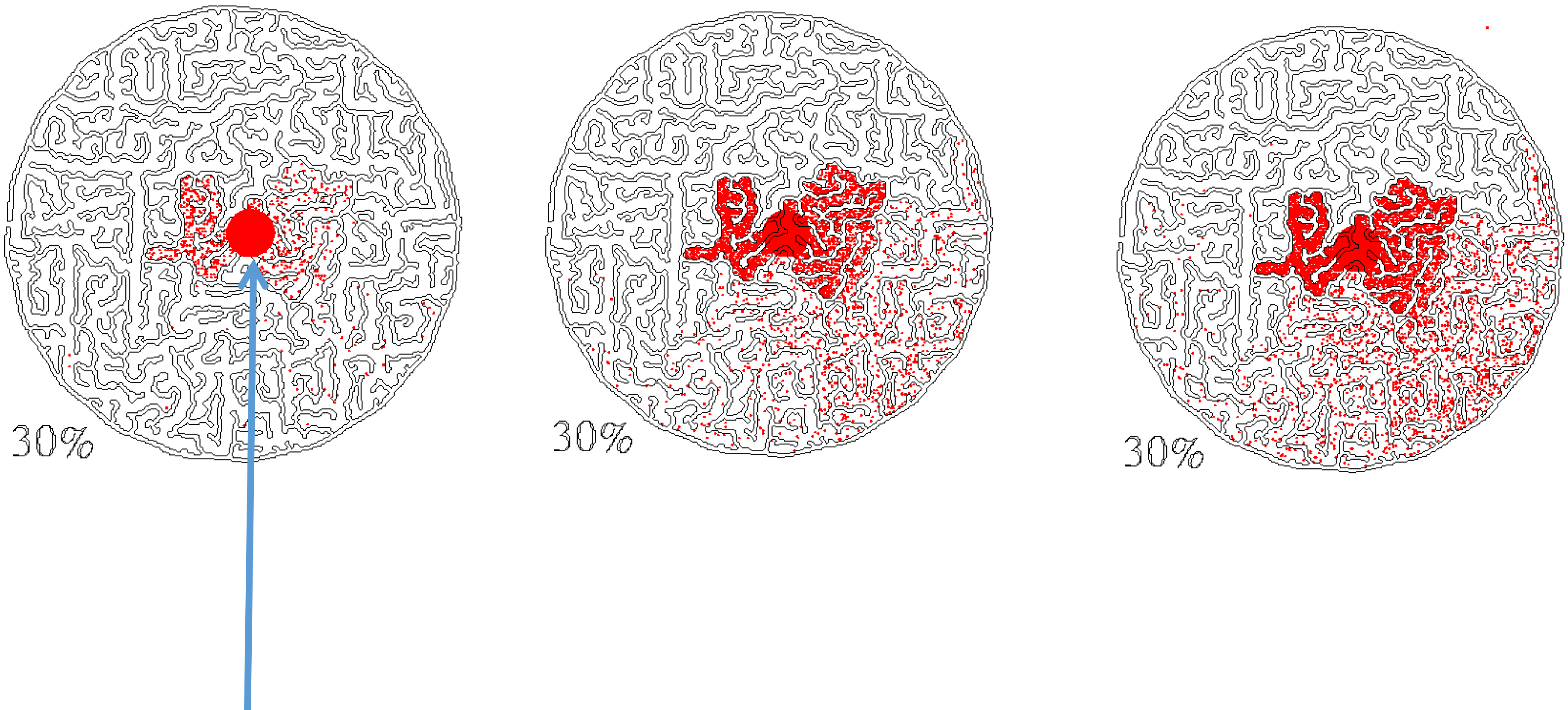
No longer $\tau \ll t_c = l_b/(2D)$

Diffusion on land or water -- comparison



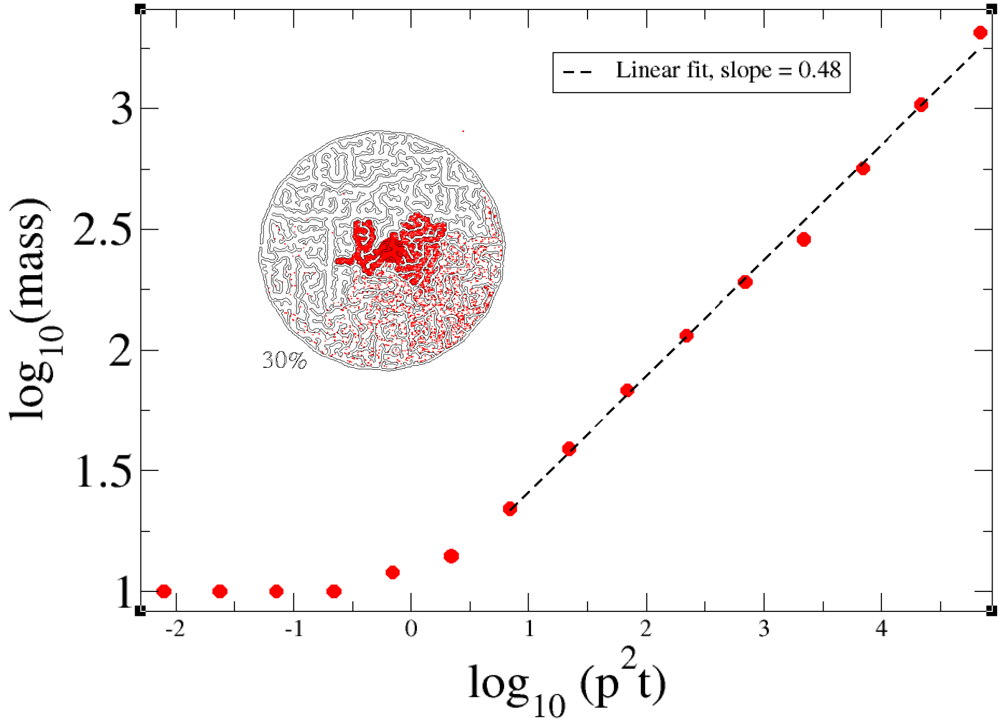


Steady heating by constant C in center – a different diffusion problem



A fixed number 10 random walkers is maintained in the small central disk

Maintaining a constant concentration in the center

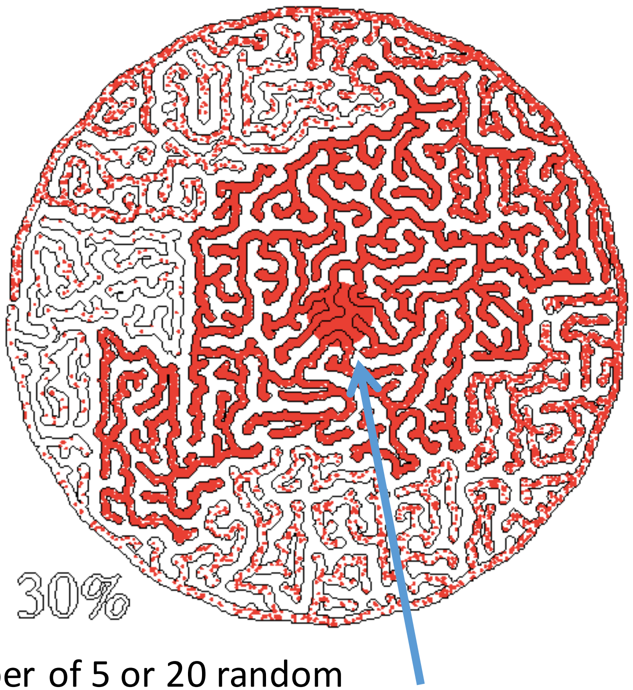


Transport rates:

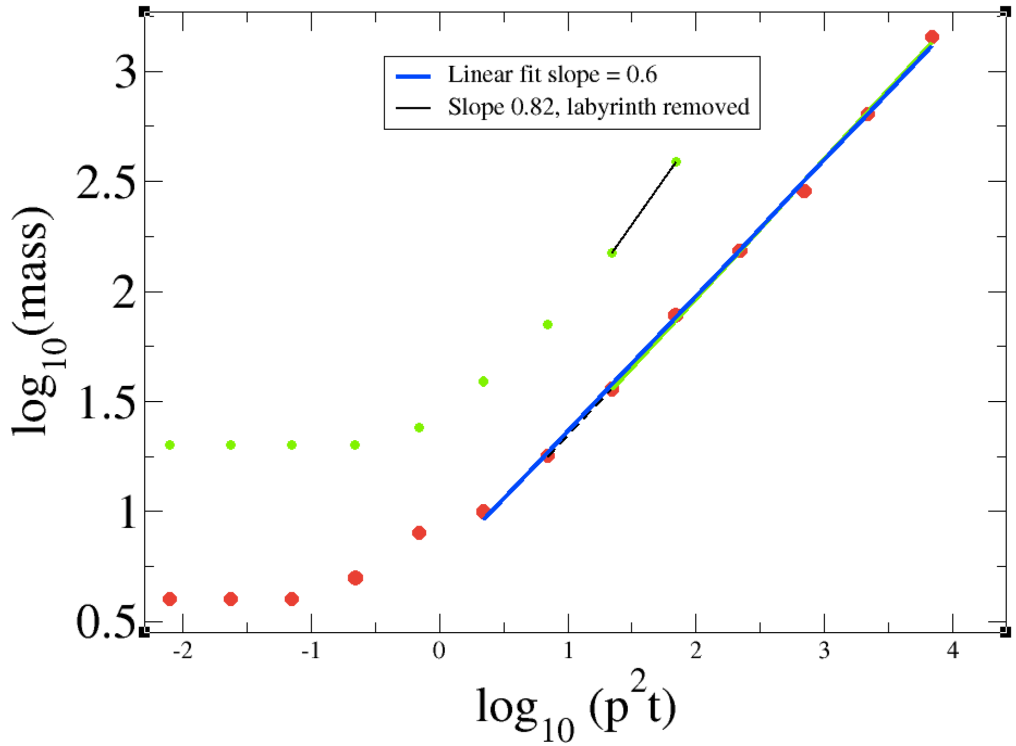
$$\dot{M} \propto C_0 \begin{cases} \frac{1}{\sqrt{t}} & \text{absorbed in a 2D labyrinth} \\ \frac{1}{\ln(t/a)} & \text{in 2D open space} \\ \frac{1}{r_c} & \text{in 3D open space} \end{cases}$$

Delayed absorption or desorption – a potentially useful property for controlled drug- release.

Steady heating by constant C in center, diffusion on land



A fixed number of 5 or 20 random walkers is maintained in the small central disk



Summary

- Diffusivity characterizes geometry in a non-trivial way
- Structures quickly enter regime of anomalous diffusion ($\tau \ll t_c = l_b/(2D)$)
- Experimentally feasible

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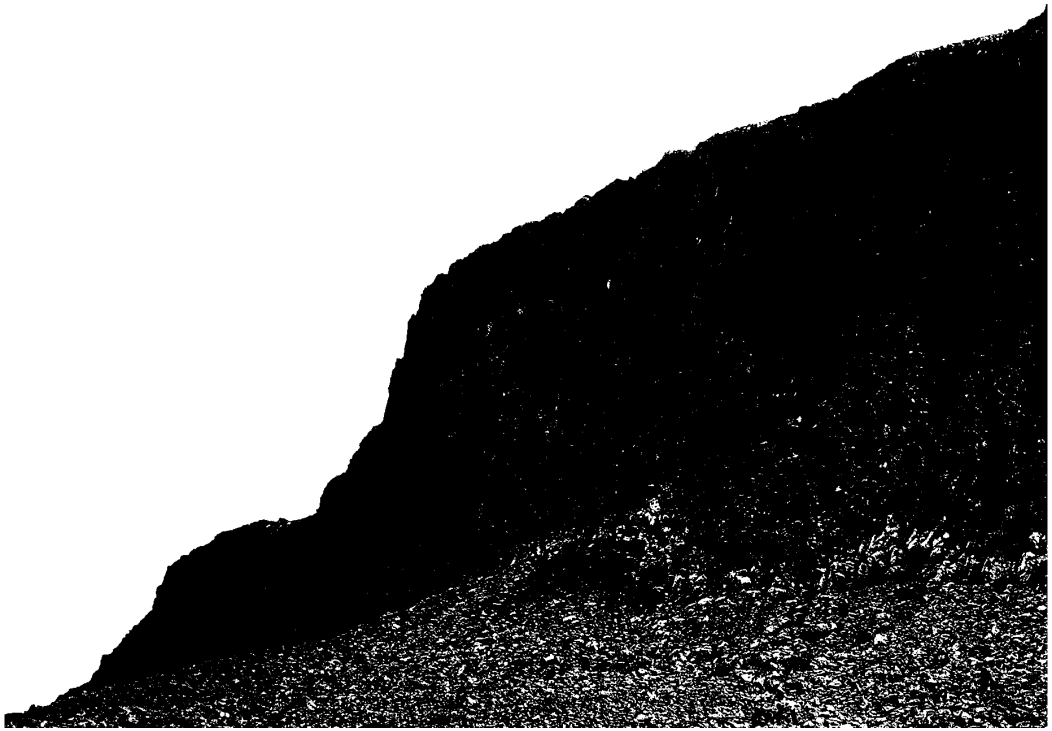
Diffusion on a boulder field



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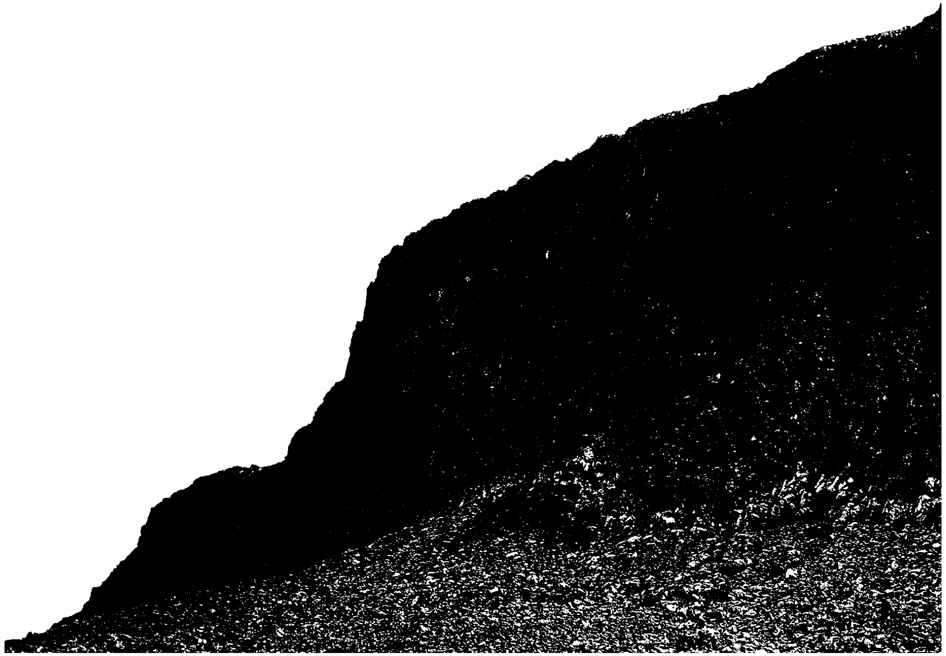

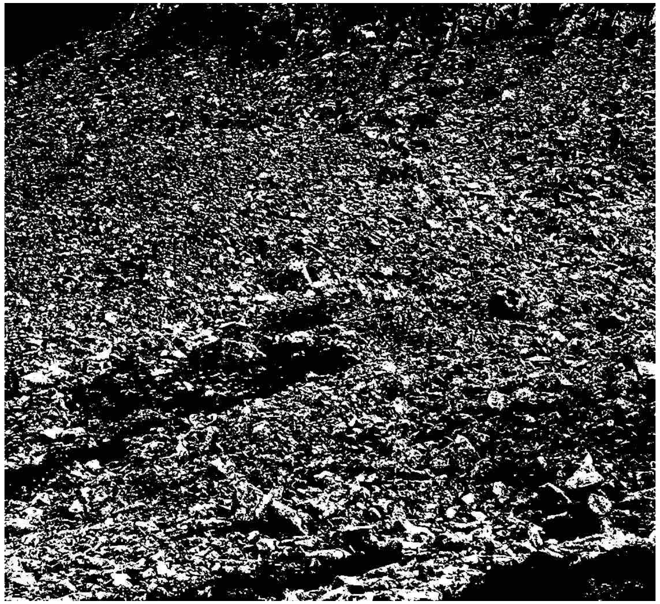

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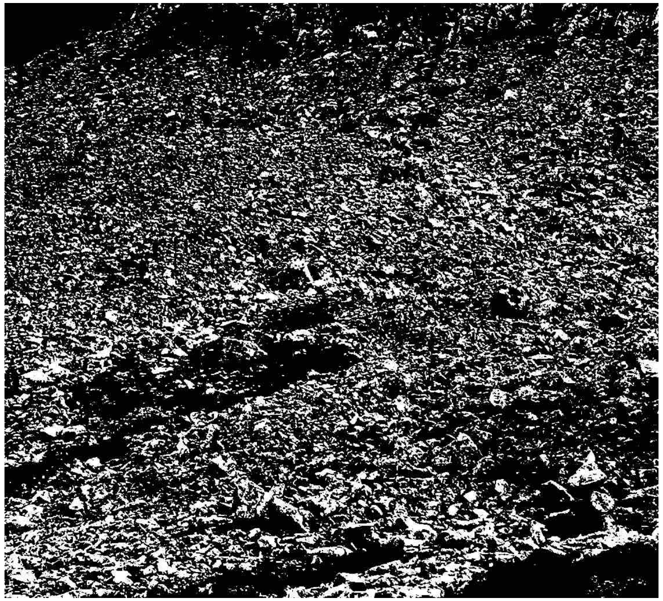

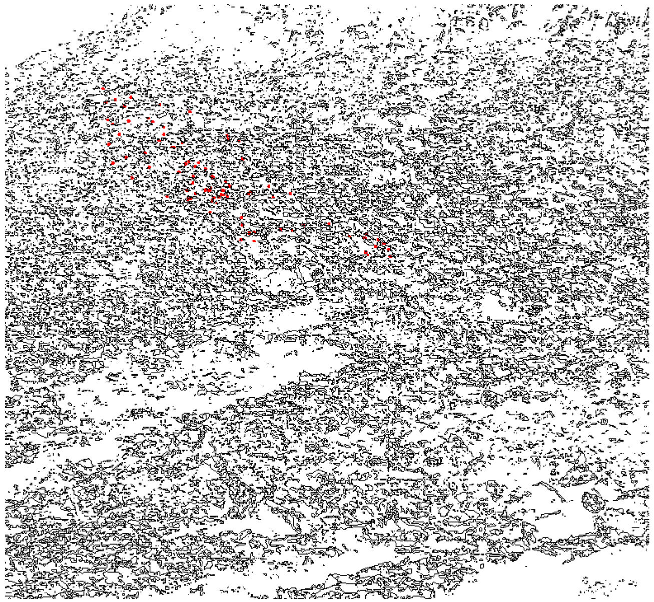

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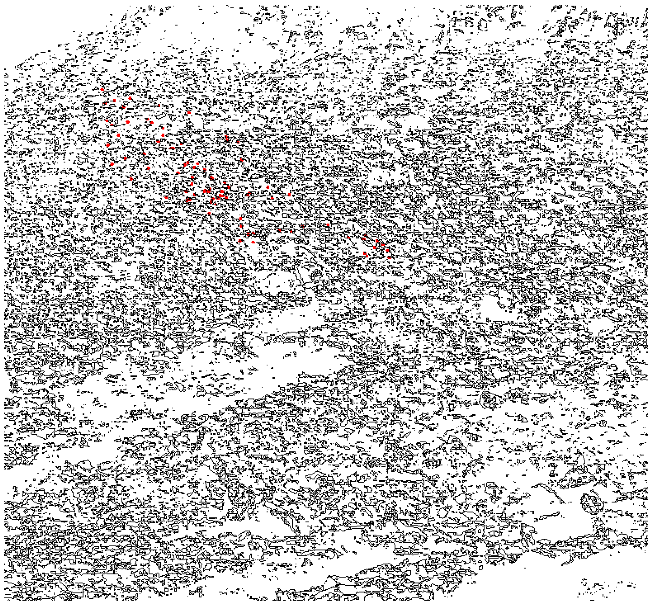
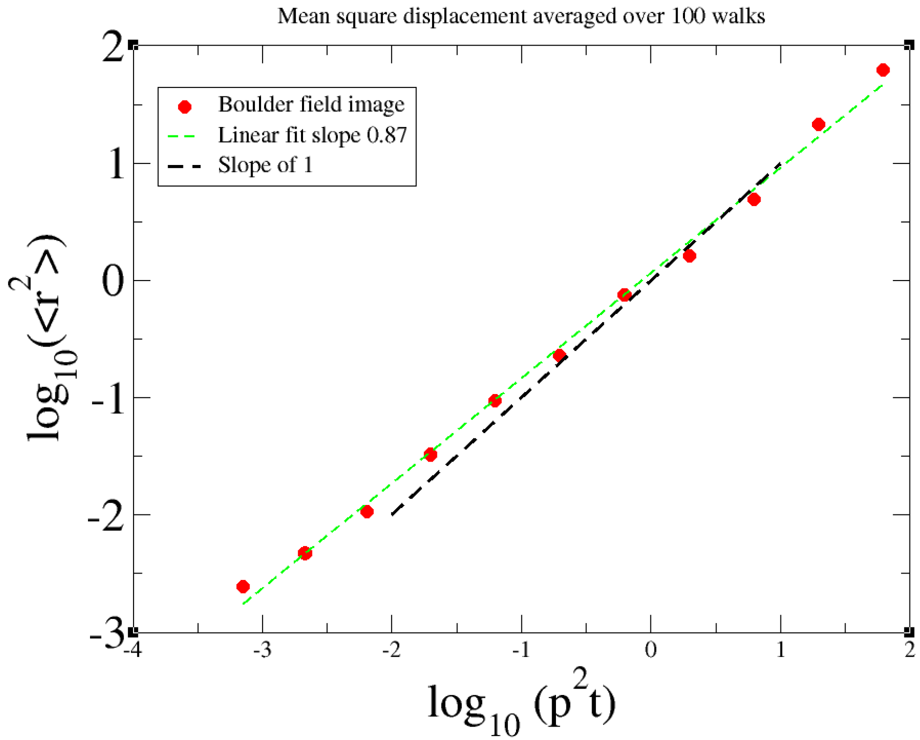
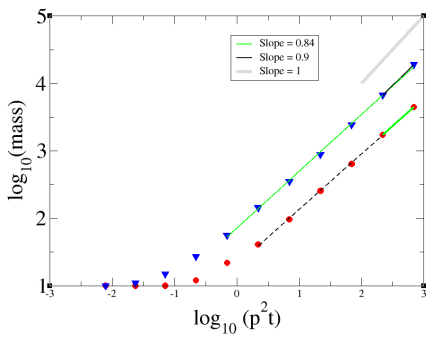
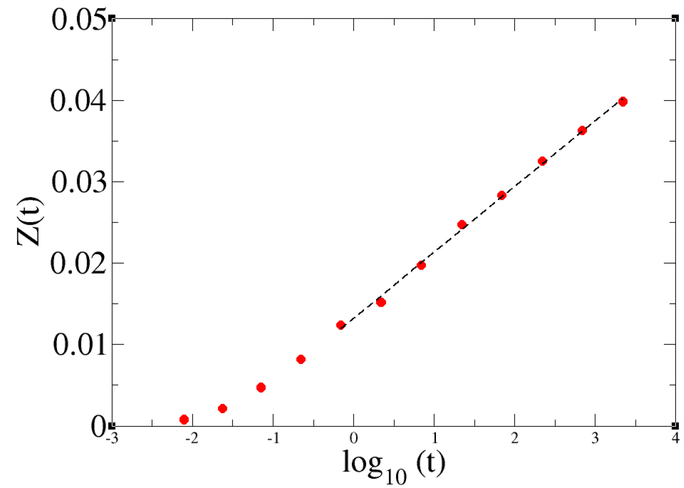




Steady heating by constant C in center, diffusion without boundaries





A fixed number of 10 random walkers is maintained in a small central disk of variable size,

Making the ansatz that the mass

$$M(t) = B\frac{t}{\log_{10}(t/a)} \tag{3}$$

implies that

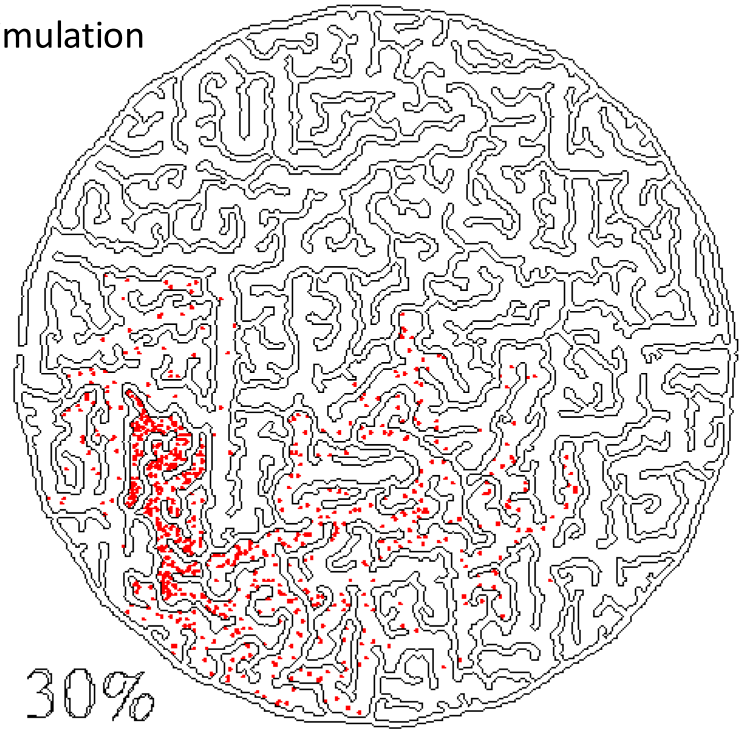
$$Z(t) = 10^{\log_{10}(t)-\log_{10}(M)} = \frac{\log_{10}(t)-\log_{10}(a)}{B} \tag{4}$$

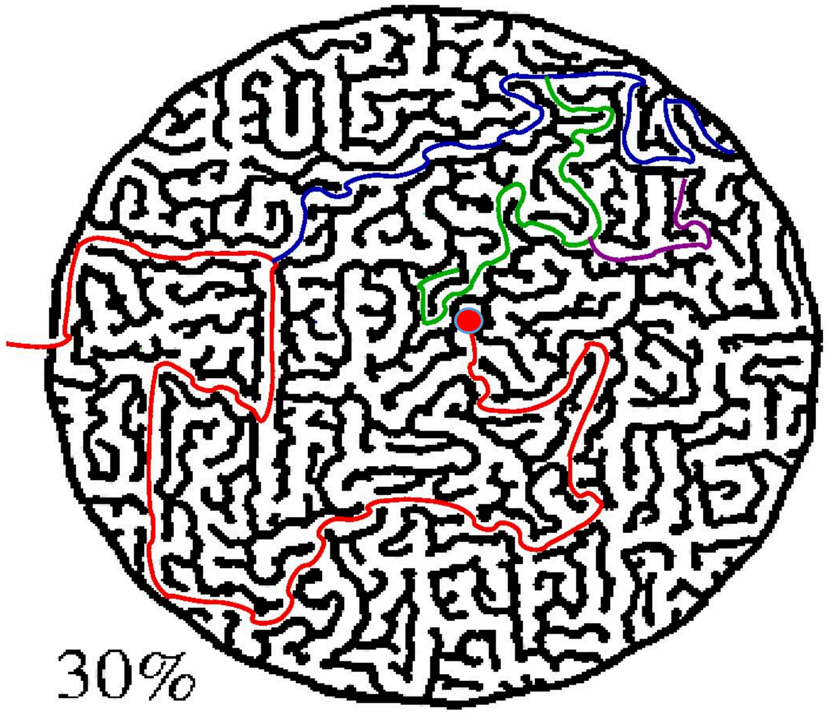
which is indeed seen to hold with $B \approx$125 and $a \approx 0.025$ s s. The time t_n is in units that are linked to the step length of the random walker, and $t_n = 9\ 10^{-4}n$ s where n is the integer time step. The only time-scale available in the problem is the one given by the inner circle radius $r_c = 3$ mm, and the diffusivity $D = (1/2)\text{cm}^2/2$, i.e. $\tau = 2r_c^2/D =$ 0.09 s which is of the same order of magnitude as a

The patterns

Backbone and dead ends of several generations

Simulation





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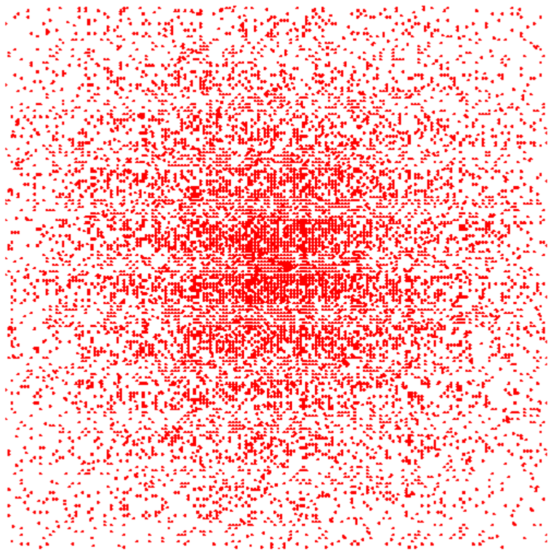

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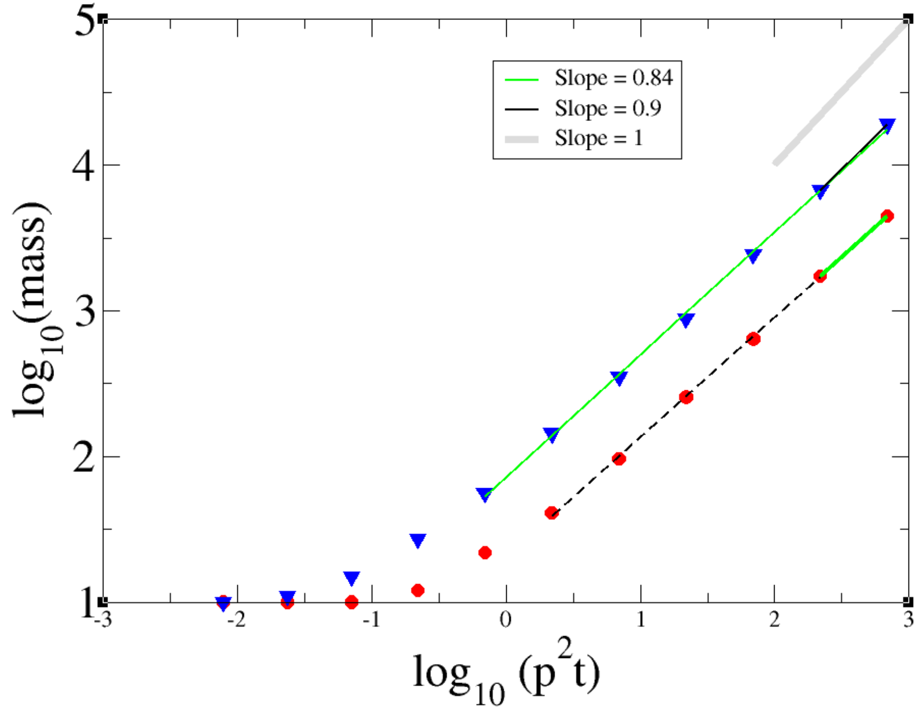

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