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Senter for fremragende forskning
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Discontinuous approach for modelling distinct ice lenses in frost heave phenomenon

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What is frost heave?

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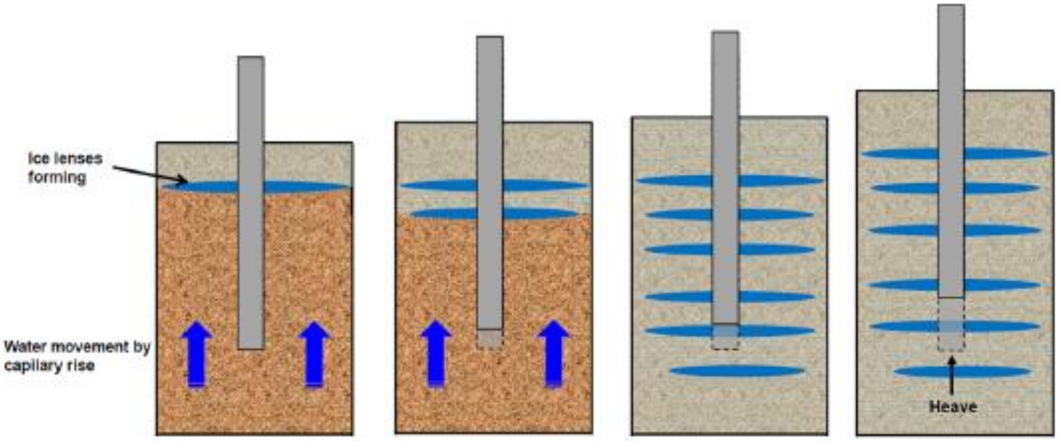
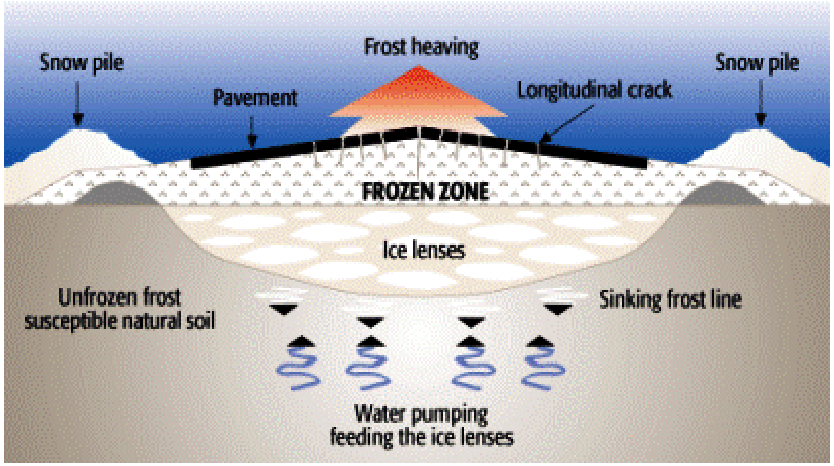
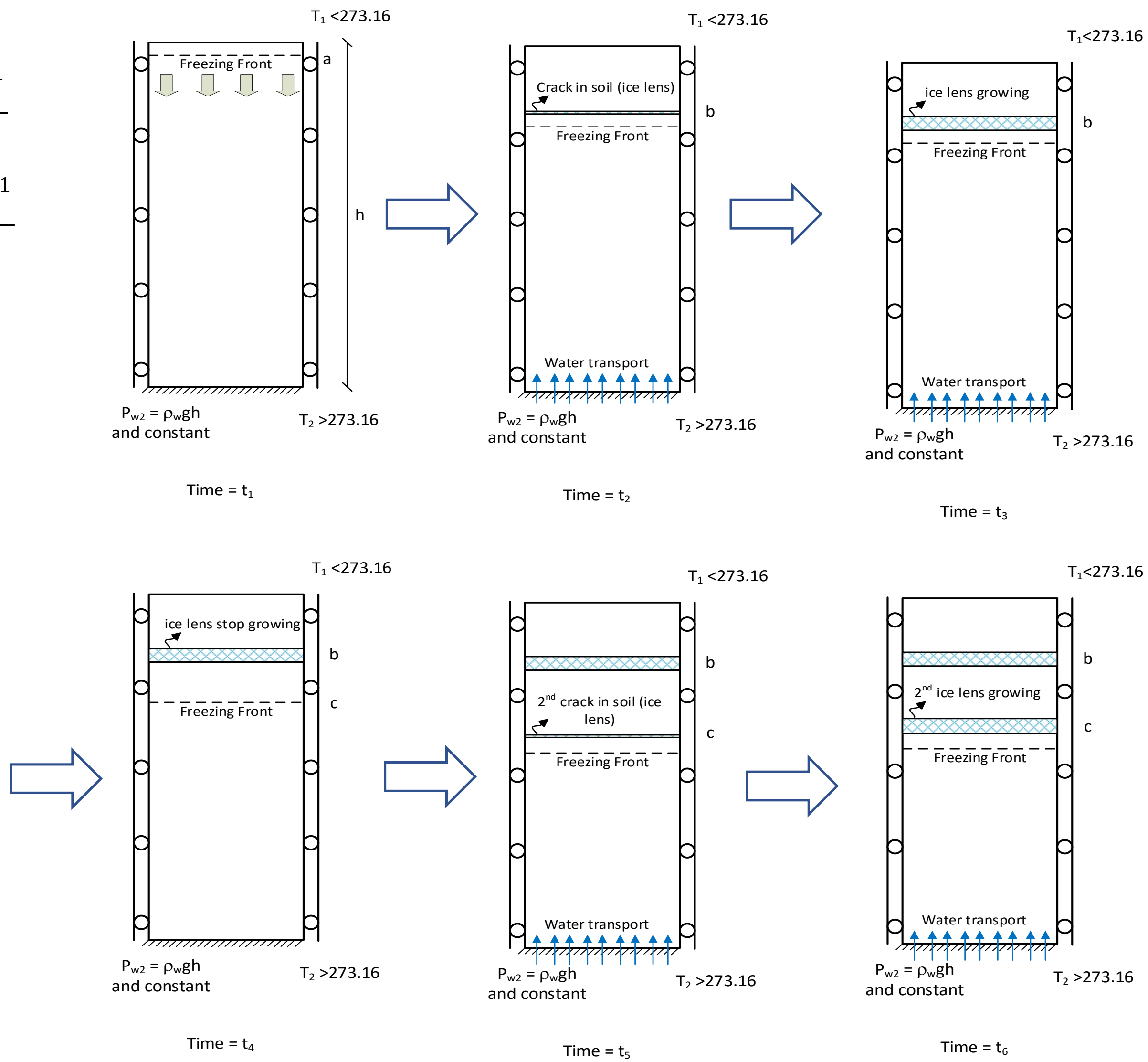


Schematic of ice lens formation and frost heave development (Ingeman-Nielsen, 2017)



Modelling Concept

$$\begin{cases} \boldsymbol{q}_w = L_{11}\dfrac{\partial p_w}{\partial x} + L_{12}\dfrac{\partial T^{-1}}{\partial x} \\ \boldsymbol{q}_H = L_{12}\dfrac{\partial p_w}{\partial x} + L_{12}\dfrac{\partial T^{-1}}{\partial x} \end{cases}$$



Coefficients

$$\begin{cases} \boldsymbol{q}_w = -L_{11}(\nabla p_w - \rho_w \boldsymbol{g}) + L_{12}\nabla T^{-1} \\ \\ \boldsymbol{q}_H = L_{12}(\nabla p_w - \rho_w \boldsymbol{g}) + L_{22}\nabla T^{-1} \end{cases}$$

$$L_{11} = \frac{k_r K}{\mu_w} \quad ; \text{where} \quad k_r = \sqrt{s_w}\left[1-\left(1-s_w^{1/\lambda}\right)^{\lambda}\right]^2$$

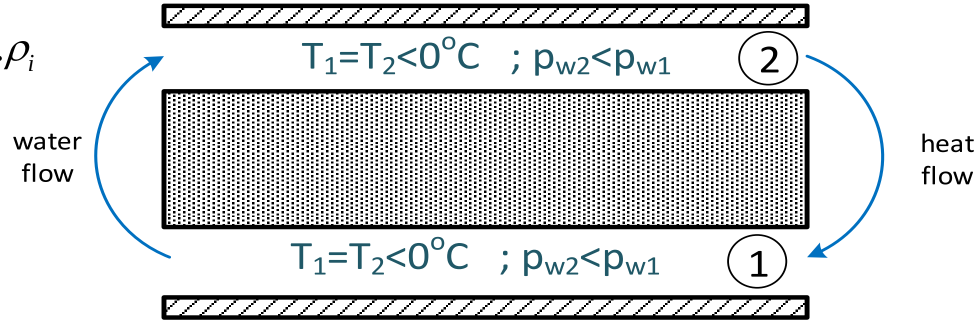
$$L_{22} = T^2\left(\lambda_s^{1-n}.\lambda_w^{ns_w}.\lambda_i^{ns_i}\right)$$

For the coupling coefficient L_{12}:

From energy balance: $q_H = -q_w.l.\rho_i = L_{11}\nabla(p_w).l.\rho_i$

From Transport equation: $q_H = L_{12}\nabla(p_w)$

$$\Rightarrow \quad L_{12} = L_{11}.l.\rho_i$$



No gravity effect

Mass Transfer

$$n\left(\rho_w-\rho_i\right)\frac{D^s s_w}{Dt}+\nabla.\left(\rho_w n s_w \boldsymbol{w}_w\right)+\left(s_w\rho_w+s_i\rho_i\right)\nabla.\boldsymbol{v}_s=0$$

$$s_w=s_w(T)$$

$$\nabla.\boldsymbol{v}_s=-\dot{\varepsilon}_v$$

$$n s_w \boldsymbol{w}_w=\boldsymbol{q}_w=-L_{11}(\nabla p_w-\rho_w \boldsymbol{g})+L_{12}\nabla T^{-1}$$

$$\left(\rho C\right)_{\text{eff}}\frac{D^sT}{Dt}+\left(\rho_w C_w \boldsymbol{q}_w\right).\nabla T+\nabla.\boldsymbol{q}_H-l.\dot{m}_{w\to i}=0$$

$$\left(\rho C\right)_{\text{eff}}=(1-n)\rho_s C_s+ns_w C_w+ns_i C_i$$

$$\boldsymbol{q}_H=L_{12}(\nabla p_w-\rho_w \boldsymbol{g})+L_{22}\nabla T^{-1}$$

ice mass balance:

$$-n\rho_i\frac{D^s s_w}{Dt}+s_i\rho_i\nabla.\boldsymbol{v}_s-\dot{m}_{w\to i}=0$$

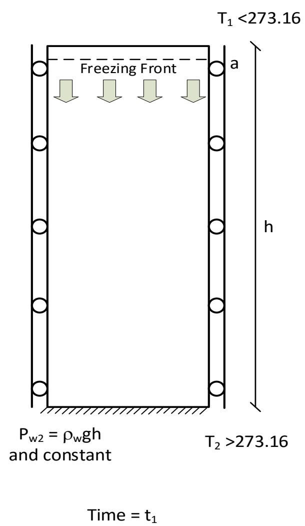
$$\Rightarrow \dot{m}_{w\to i}=-n\rho_i\frac{D^s s_w}{Dt}+s_i\rho_i\nabla.\boldsymbol{v}_s$$

Force Equilibrium

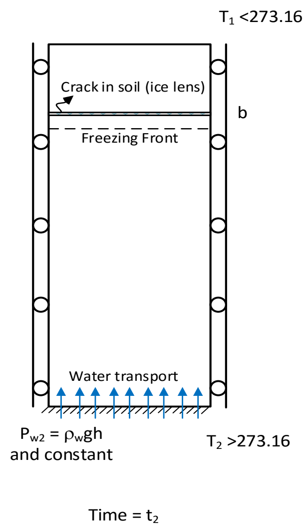
$$\nabla.\boldsymbol{\sigma} + \rho \boldsymbol{g} = 0$$

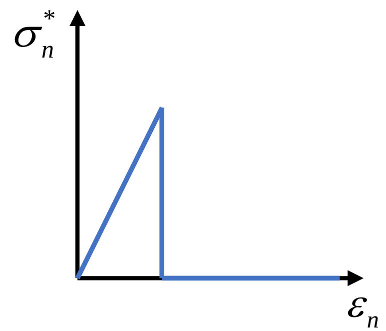
$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^* + p_w \boldsymbol{I}$$

$$d\boldsymbol{\sigma}^* = \boldsymbol{D}^e . d\boldsymbol{\varepsilon}$$



Crack Criterion





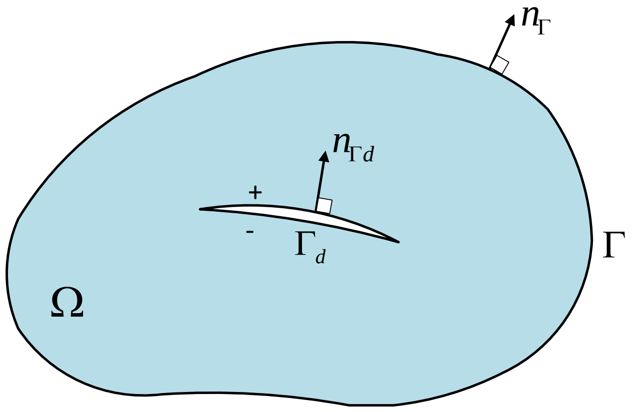
Assumption:
Crack will appear normal to temperature gradient.

Continuum domain: water pressure will increase to cancel the temperature induced flux.

Discontinuous domain:
If water is available, water pressure will release and swelling will start.

Weak form of Continuity Eq.

$$n\left(\rho_w-\rho_i\right)\frac{D^s s_w}{Dt}+\nabla.\left(\rho_w n s_w \boldsymbol{w}_w\right)+\left(s_w\rho_w+s_i\rho_i\right)\nabla.\boldsymbol{v}_s=0$$



$$\int_\Omega \delta p_w\left[n\left(\rho_w-\rho_i\right)\frac{D^s s_w}{Dt}\right]d\Omega-\int_\Omega \nabla\left(\delta p_w\right).\left(\rho_w \boldsymbol{q}_w\right)d\Omega-\int_{\Gamma_d}\delta p_w.\rho_w \bar{q}_{w_d}\,d\Gamma$$
$$+\int_{\Gamma_{qw}}\delta p_w.\rho_w\bar{q}_w\,d\Gamma-\int_\Omega \delta p_w\left[\left(s_w\rho_w+s_i\rho_i\right)\dot{\varepsilon}_v\right]d\Omega=0$$

Could be calculated by writing continuity equation in the crack

Assumptions:

- Water pressure is continuous
- Water flux is discontinuous; i.e. there is a jump

Weak form of Continuity Eq. in the Crack

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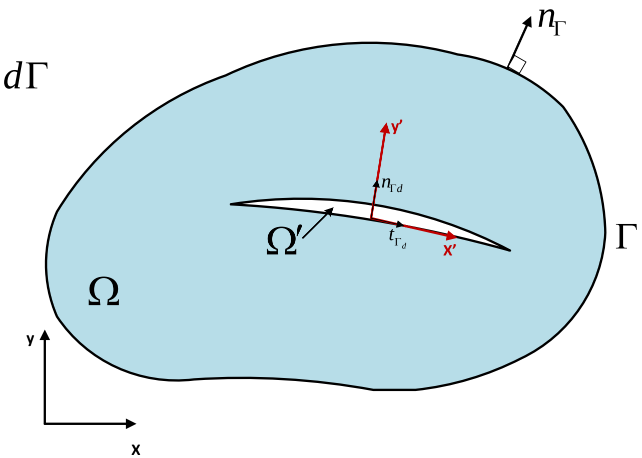
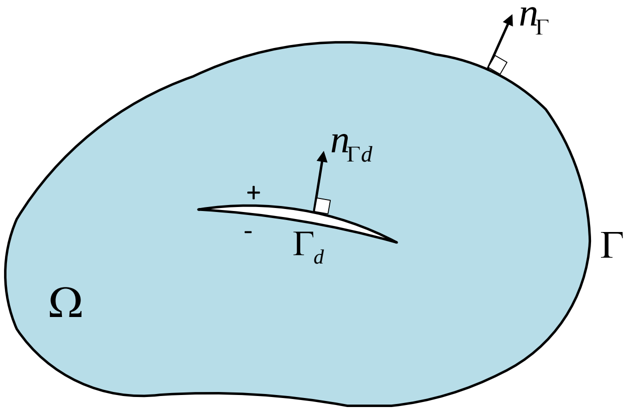
$$\int_{\Omega'} \delta p_w \left[(\rho_w - \rho_i) \frac{D^s s_{w_d}}{Dt} \right] d\Omega - \int_{\Omega'} \nabla(\delta p_w).(\rho_w \boldsymbol{q}_w) d\Omega + \int_{\Gamma_d} \delta p_w . \rho_w \bar{q}_{w_d} d\Gamma$$
$$+ \int_{\Omega'} \delta p_w \left[\left(s_{w_d} \rho_w + s_{i_d} \rho_i \right) \nabla . \boldsymbol{v}_s \right] d\Omega = 0$$



$$\Rightarrow \int_{\Gamma_d} \delta p_w . \rho_w \bar{q}_{w_d} d\Gamma = \underbrace{- \int_{\Omega'} \delta p_w \left[(\rho_w - \rho_i) \frac{D^s s_{w_d}}{Dt} \right] d\Omega}_{\text{Integral I}} + \underbrace{\int_{\Omega'} \nabla(\delta p_w).(\rho_w \boldsymbol{q}_w) d\Omega}_{\text{Integral II}}$$
$$\underbrace{- \int_{\Omega'} \delta p_w \left[\left(s_{w_d} \rho_w + s_{i_d} \rho_i \right) \nabla . \boldsymbol{v}_s \right] d\Omega}_{\text{Integral III}}$$

Weak form of Heat Transfer Eq.

$$(\rho C)_{\text{eff}} \frac{D^s T}{Dt} + (\rho_w C_w \boldsymbol{q}_w).\nabla T + \nabla.\boldsymbol{q}_H - l.\dot{m}_{w\to i} = 0$$



$$\int_\Omega \delta T (\rho C)_{\text{eff}} \frac{D^s T}{Dt}\, d\Omega + \int_\Omega \delta T (\rho_w C_w \boldsymbol{q}_w).\nabla T\, d\Omega - \int_\Omega \nabla(\delta T).\boldsymbol{q}_H\, d\Omega -$$
$$\int_{\Gamma_d} \delta T\, \bar{q}_{H_d} d\Gamma + \int_\Gamma \delta T\, \bar{q}_H d\Gamma - \int_\Omega \delta T. l.\dot{m}_{w\to i}\, d\Omega = 0$$

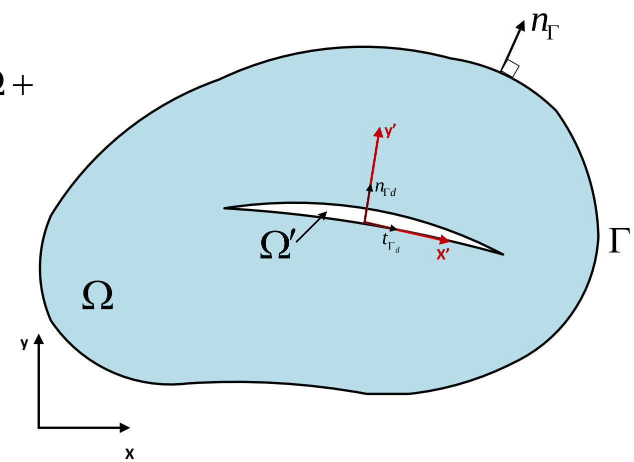
Could be calculated by writing heat transfer equation in the crack

Assumptions:

- Temperature is continuous
- Temperature flux is discontinuous; i.e. there is a jump

Weak form of Heat Transfer Eq. in the Crack

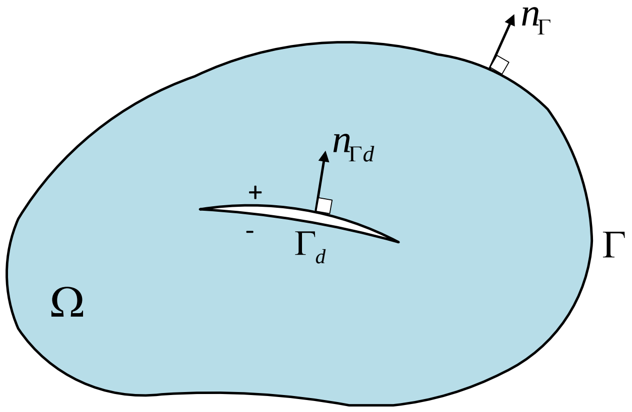
$$\int_{\Omega'} \delta T \left(\rho C\right)_{\mathrm{eff}_d} \frac{D^s T}{Dt}\, d\Omega + \int_{\Omega'} \delta T \left(\rho_w C_w \boldsymbol{q}_w\right).\nabla T \; d\Omega - \int_{\Omega'} \nabla\left(\delta T\right).\boldsymbol{q}_H \; d\Omega + \int_{\Gamma_d} \delta T\, \bar{q}_{H_d}\, d\Gamma - \int_{\Omega'} \delta T.l.\left(\dot{m}_{w\to i}\right)_d d\Omega = 0$$



$$\Rightarrow \quad \int_{\Gamma_d} \delta T\, \bar{q}_{H_d}\, d\Gamma = -\underbrace{\int_{\Omega'} \delta T \left(\rho C\right)_{\mathrm{eff}_d} \frac{D^s T}{Dt}\, d\Omega}_{\text{Integral I}} - \underbrace{\int_{\Omega'} \delta T \left(\rho_w C_w \boldsymbol{q}_w\right).\nabla T \; d\Omega}_{\text{Integral II}} + \underbrace{\int_{\Omega'} \nabla\left(\delta T\right).\boldsymbol{q}_H \; d\Omega}_{\text{Integral III}} + \underbrace{\int_{\Omega'} \delta T.l\left(\dot{m}_{w\to i}\right)_d d\Omega}_{\text{Integral IV}}$$

Weak form of Equilibrium Equation

$$\nabla.\boldsymbol{\sigma} - \rho \boldsymbol{g} = 0$$



$$-\int_\Omega \nabla(\delta \boldsymbol{u}) : \boldsymbol{\sigma}^* d\Omega - \int_\Omega \nabla(\delta \boldsymbol{u}) : (p_w \boldsymbol{I}) d\Omega -$$
$$\int_{\Gamma_d} \delta \boldsymbol{u} \, .(p_w \boldsymbol{I}) \boldsymbol{n}_{\Gamma_d} d\Gamma + \int_{\Gamma_d} \delta \boldsymbol{u} \, .(\bar{\boldsymbol{t}}_d \boldsymbol{I}) \boldsymbol{n}_{\Gamma_d} d\Gamma \int_{\Gamma_t} \delta \boldsymbol{u}.\bar{\boldsymbol{t}} \, d\Gamma - \int_\Omega \delta \boldsymbol{u}.\rho \boldsymbol{g} d\Omega = 0$$

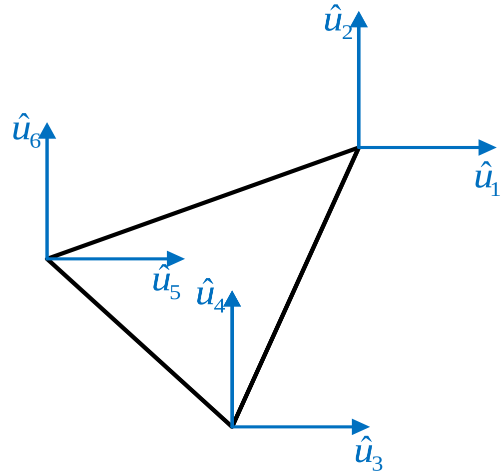
Assumptions:

- Traction is continuous
- Displacement is discontinuous; i.e. there is a jump

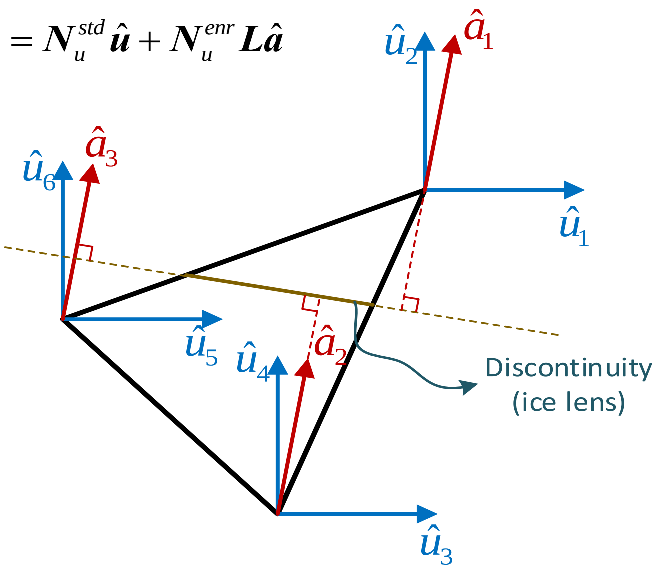
Approximation of the primary variables

Displacement field:

$$\boldsymbol{u}(\boldsymbol{x},t)=\sum_{i=1}^{N} N_{u_i}(\xi)\hat{\boldsymbol{u}}_i(t)$$



$$\boldsymbol{u}(\boldsymbol{x},t)=\sum_{i=1}^{N} N_{u_i}(\xi)\hat{\boldsymbol{u}}_i(t)+\sum_{j=1}^{M}\left[\bar{N}_{u_j}(\xi)\Big(H(\varphi(\xi))-H(\varphi(\xi_j))\Big)\boldsymbol{n}_{\Gamma_d}\right]\hat{a}_j(t)=\boldsymbol{N}_u^{std}\hat{\boldsymbol{u}}+\boldsymbol{N}_u^{enr}\boldsymbol{L}\hat{\boldsymbol{a}}$$

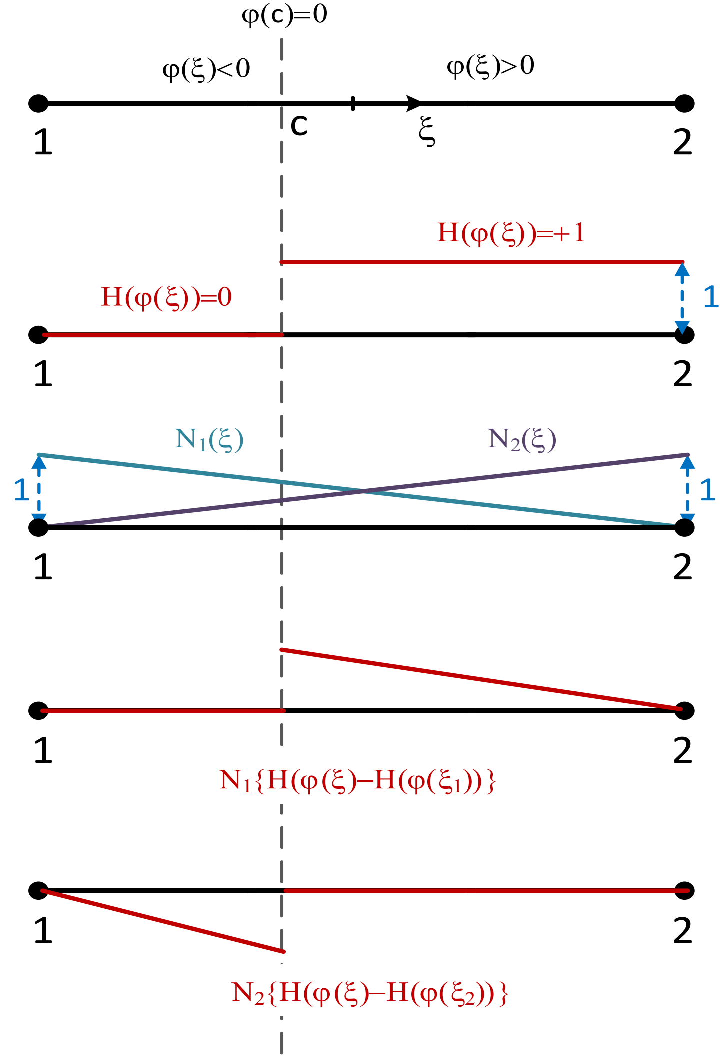


Approximation of the primary variables

Displacement field:

$$\boldsymbol{u}(\boldsymbol{x},t) = \sum_{i=1}^{N} N_{u_i}(\xi)\hat{\boldsymbol{u}}_i(t) +$$

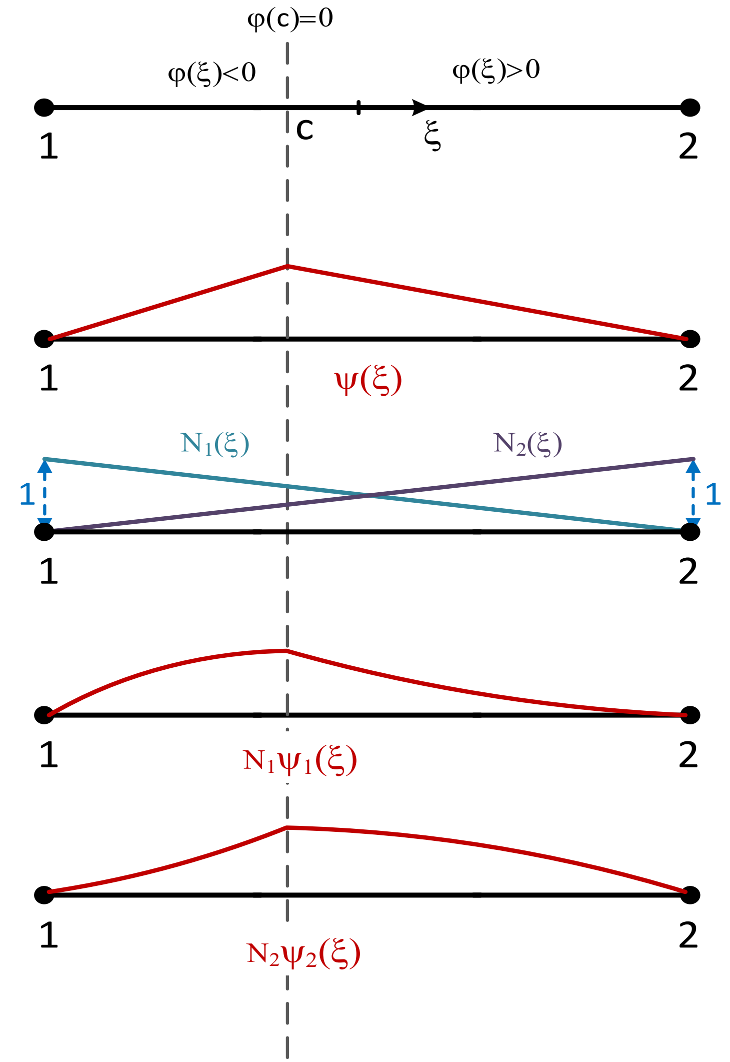
$$\sum_{j=1}^{M}\left[\bar{N}_{u_j}(\xi)\Big(H(\varphi(\xi)) - H(\varphi(\xi_j))\Big)\boldsymbol{n}_{\Gamma_d}\right]\hat{a}_j(t) = \boldsymbol{N}_u^{std}\hat{\boldsymbol{u}} + \boldsymbol{N}_u^{enr}\boldsymbol{L}\hat{\boldsymbol{a}}$$



Pressure field:

$$p_w(\boldsymbol{x},t)=\sum_{i=1}^{N} N_{p_i}(\xi)\,\hat{p}_{w_i}(t)+$$

$$\sum_{j=1}^{M}\bar{N}_{p_j}(\xi)\psi_p(\xi)\hat{b}_j(t)=\boldsymbol{N}_p^{std}\hat{\boldsymbol{p}}_w+\boldsymbol{N}_p^{enr}\hat{\boldsymbol{b}}$$



Approximation of the primary variables

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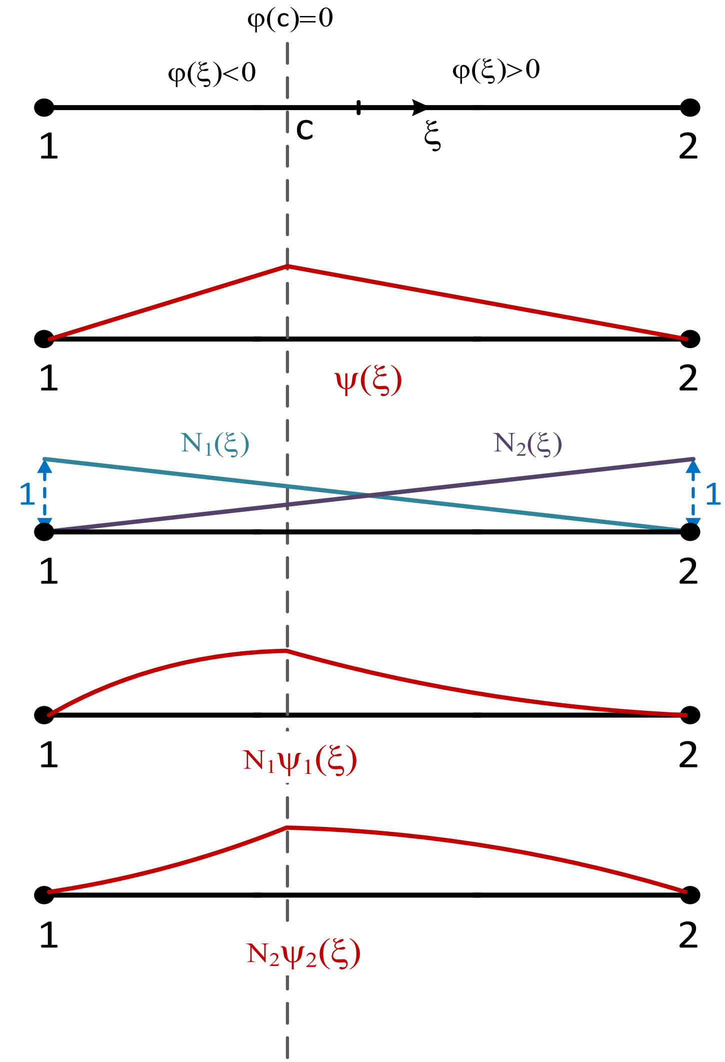
Temperature field:

$$T(\boldsymbol{x},t) = \sum_{i=1}^{N} N_{T_i}(\xi)\hat{T}_i(t) +$$

$$\sum_{j=1}^{M} \bar{N}_{T_j}(\xi)\psi_T(\xi)\hat{c}_j(t) = \boldsymbol{N}_T^{std}\hat{\boldsymbol{T}} + \boldsymbol{N}_T^{enr}\hat{\boldsymbol{c}}$$



XFEM formulation of the system

$$
\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{ua} & \boldsymbol{C}_{up} & \boldsymbol{C}_{ub} & 0 & 0 \\ \boldsymbol{K}_{au} & \boldsymbol{K}_{aa} & \boldsymbol{C}_{ap} & \boldsymbol{C}_{ab} & 0 & 0 \\ 0 & 0 & \boldsymbol{Q}_{pp} & \boldsymbol{Q}_{pb} & \boldsymbol{C}_{pT} & \boldsymbol{C}_{pc} \\ 0 & 0 & \boldsymbol{Q}_{bp} & \boldsymbol{Q}_{bb} & \boldsymbol{C}_{bT} & \boldsymbol{C}_{bc} \\ 0 & 0 & \boldsymbol{C}_{Tp} & \boldsymbol{C}_{Tb} & \boldsymbol{H}_{TT} & \boldsymbol{H}_{Tc} \\ 0 & 0 & \boldsymbol{C}_{cp} & \boldsymbol{C}_{cb} & \boldsymbol{H}_{cT} & \boldsymbol{H}_{cc} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{a}} \\ \hat{\boldsymbol{p}}_w \\ \hat{\boldsymbol{b}} \\ \hat{\boldsymbol{T}} \\ \hat{\boldsymbol{c}} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\boldsymbol{C}}_{pu} & \bar{\boldsymbol{C}}_{pa} & 0 & 0 & \bar{\boldsymbol{C}}_{pT} & \bar{\boldsymbol{C}}_{pc} \\ \bar{\boldsymbol{C}}_{bu} & \bar{\boldsymbol{C}}_{ba} & 0 & 0 & \bar{\boldsymbol{C}}_{bT} & \bar{\boldsymbol{C}}_{bc} \\ 0 & 0 & 0 & 0 & \bar{\boldsymbol{H}}_{TT} & \bar{\boldsymbol{H}}_{Tc} \\ 0 & 0 & 0 & 0 & \bar{\boldsymbol{H}}_{cT} & \bar{\boldsymbol{H}}_{cc} \end{bmatrix} \begin{bmatrix} \dot{\hat{\boldsymbol{u}}} \\ \dot{\hat{\boldsymbol{a}}} \\ \dot{\hat{\boldsymbol{p}}}_w \\ \dot{\hat{\boldsymbol{b}}} \\ \dot{\hat{\boldsymbol{T}}} \\ \dot{\hat{\boldsymbol{c}}} \end{bmatrix} - \begin{bmatrix} 0 \\ \boldsymbol{F}_a^{\text{int}} \\ \boldsymbol{F}_p^{\text{int}} \\ \boldsymbol{F}_b^{\text{int}} \\ \boldsymbol{F}_T^{\text{int}} \\ \boldsymbol{F}_c^{\text{int}} \end{bmatrix} - \begin{bmatrix} \boldsymbol{F}_u^{\text{ext}} \\ \boldsymbol{F}_a^{\text{ext}} \\ \boldsymbol{F}_p^{\text{ext}} \\ \boldsymbol{F}_b^{\text{ext}} \\ \boldsymbol{F}_T^{\text{ext}} \\ \boldsymbol{F}_c^{\text{ext}} \end{bmatrix} = 0
$$

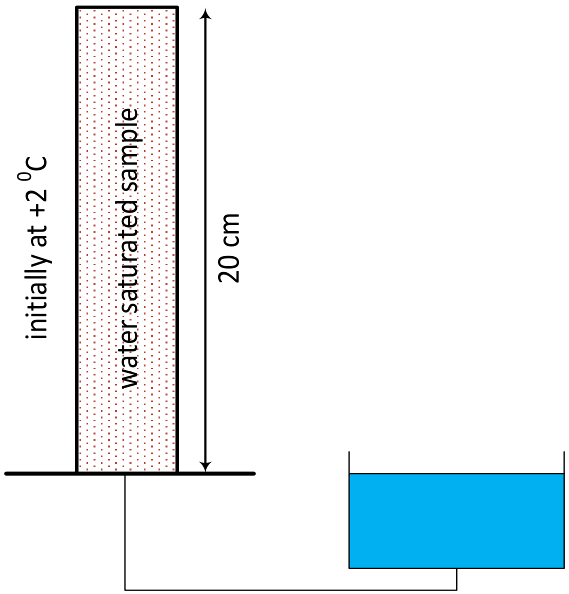
fully implicit first-order accurate finite difference scheme, and linearization:

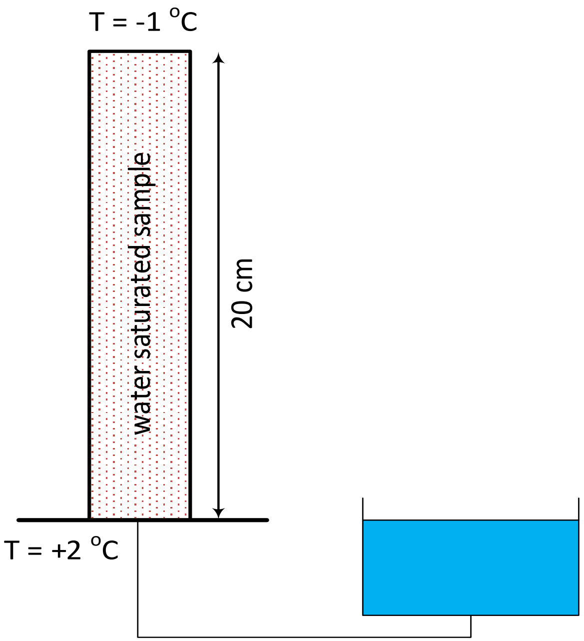
$$
\mathbf{J}_{n+1}^{i}.\Delta \boldsymbol{X}_{n+1}^{i+1} = -\mathrm{R}_{n+1}^{i}
$$

$$
\mathrm{R}_{n+1}^{i} = \begin{bmatrix} 0 & 0 & \boldsymbol{C}_{up} & \boldsymbol{C}_{ub} & 0 & 0 \\ 0 & 0 & \boldsymbol{C}_{ap} & \boldsymbol{C}_{ab} & 0 & 0 \\ \bar{\boldsymbol{C}}_{pu} & \bar{\boldsymbol{C}}_{pa} & \Delta t\boldsymbol{Q}_{pp} & \Delta t\boldsymbol{Q}_{pb} & \bar{\boldsymbol{C}}_{pT} + \Delta t\boldsymbol{C}_{pT} & \bar{\boldsymbol{C}}_{pc} + \Delta t\boldsymbol{C}_{pc} \\ \bar{\boldsymbol{C}}_{bu} & \bar{\boldsymbol{C}}_{ba} & \Delta t\boldsymbol{Q}_{bp} & \Delta t\boldsymbol{Q}_{bb} & \bar{\boldsymbol{C}}_{bT} + \Delta t\boldsymbol{C}_{bT} & \bar{\boldsymbol{C}}_{bc} + \Delta t\boldsymbol{C}_{bc} \\ 0 & 0 & \Delta t\boldsymbol{C}_{Tp} & \Delta t\boldsymbol{C}_{Tb} & \bar{\boldsymbol{H}}_{TT} + \Delta t\boldsymbol{H}_{TT} & \bar{\boldsymbol{H}}_{Tc} + \Delta t\boldsymbol{H}_{Tc} \\ 0 & 0 & \Delta t\boldsymbol{C}_{cp} & \Delta t\boldsymbol{C}_{cb} & \bar{\boldsymbol{H}}_{cT} + \Delta t\boldsymbol{H}_{cT} & \bar{\boldsymbol{H}}_{cc} + \Delta t\boldsymbol{H}_{cc} \end{bmatrix}_{n+1}^{i} \begin{bmatrix} \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{a}} \\ \hat{\boldsymbol{p}}_w \\ \hat{\boldsymbol{b}} \\ \hat{\boldsymbol{T}} \\ \hat{\boldsymbol{c}} \end{bmatrix}_{n+1}^{i} -
$$

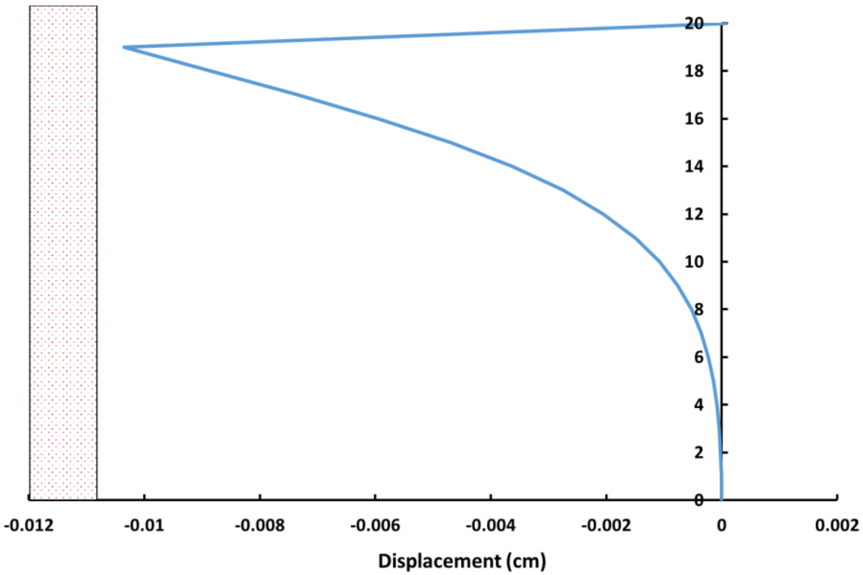
$$
\begin{bmatrix} \int_\Omega \left(\boldsymbol{B}_u^{std}\right)^T : \boldsymbol{\sigma}^* d\Omega \\ \int_\Omega \left(\boldsymbol{B}_u^{enr}\boldsymbol{L}\right)^T : \boldsymbol{\sigma}^* d\Omega \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{n+1}^{i} - \begin{bmatrix} 0 \\ \boldsymbol{F}_a^{\text{int}} \\ \Delta t\boldsymbol{F}_p^{\text{int}} \\ \Delta t\boldsymbol{F}_b^{\text{int}} \\ \Delta t\boldsymbol{F}_T^{\text{int}} \\ \Delta t\boldsymbol{F}_c^{\text{int}} \end{bmatrix}_{n+1}^{i} - \begin{bmatrix} \boldsymbol{F}_u^{\text{ext}} \\ \boldsymbol{F}_a^{\text{ext}} \\ \Delta t\boldsymbol{F}_p^{\text{ext}} \\ \Delta t\boldsymbol{F}_b^{\text{ext}} \\ \Delta t\boldsymbol{F}_T^{\text{ext}} \\ \Delta t\boldsymbol{F}_c^{\text{ext}} \end{bmatrix}_{n+1} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\boldsymbol{C}}_{pu} & \bar{\boldsymbol{C}}_{pa} & 0 & 0 & \bar{\boldsymbol{C}}_{pT} & \bar{\boldsymbol{C}}_{pc} \\ \bar{\boldsymbol{C}}_{bu} & \bar{\boldsymbol{C}}_{ba} & 0 & 0 & \bar{\boldsymbol{C}}_{bT} & \bar{\boldsymbol{C}}_{bc} \\ 0 & 0 & 0 & 0 & \bar{\boldsymbol{H}}_{TT} & \bar{\boldsymbol{H}}_{Tc} \\ 0 & 0 & 0 & 0 & \bar{\boldsymbol{H}}_{cT} & \bar{\boldsymbol{H}}_{cc} \end{bmatrix}_{n+1}^{i} \begin{bmatrix} \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{a}} \\ \hat{\boldsymbol{p}}_w \\ \hat{\boldsymbol{b}} \\ \hat{\boldsymbol{T}} \\ \hat{\boldsymbol{c}} \end{bmatrix}_{n}
$$

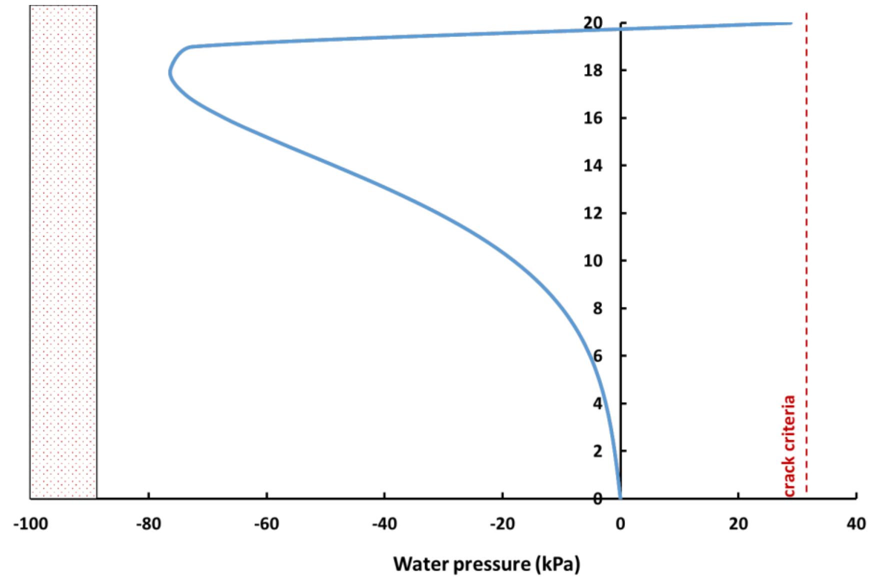
1-Dimensional Test

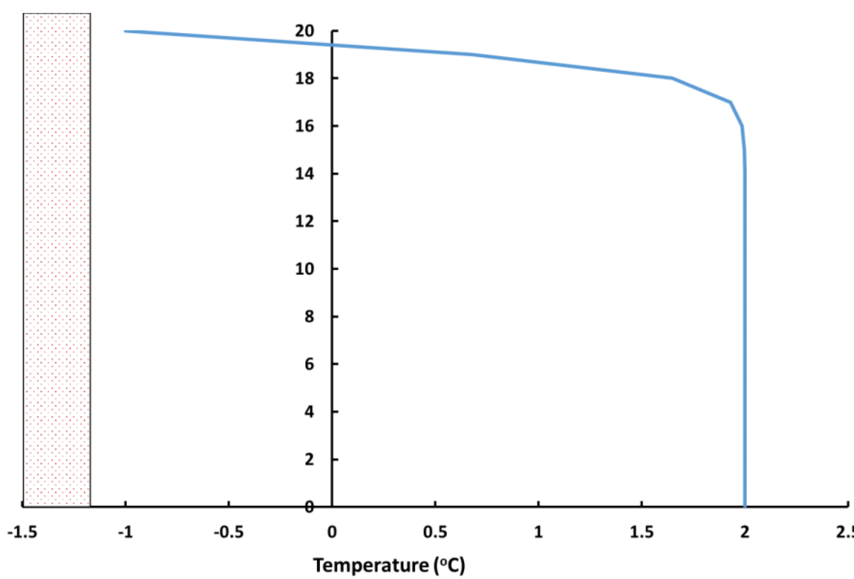




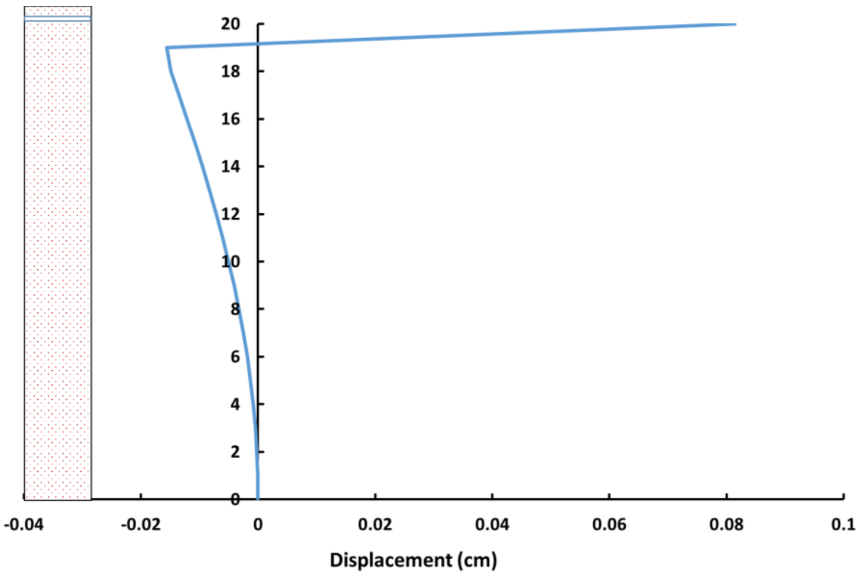
1-Dimensional Test (1st ice lens)

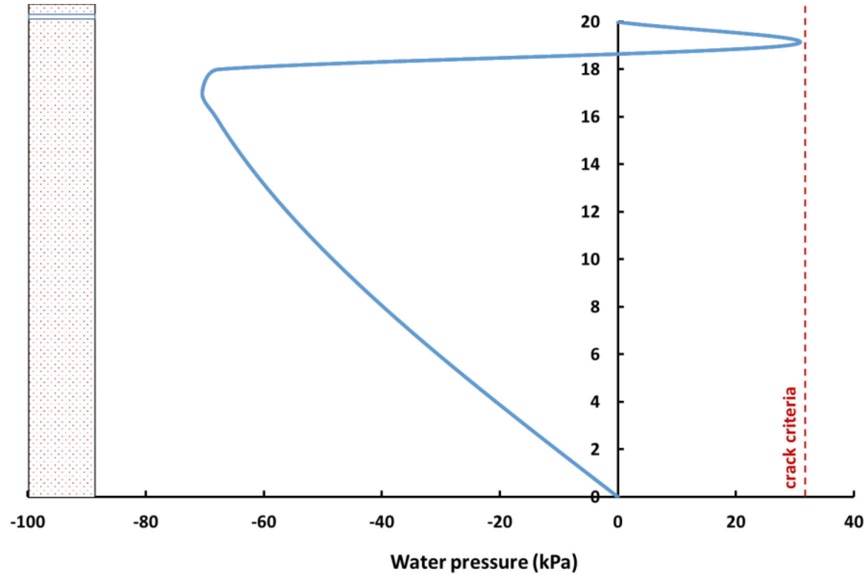


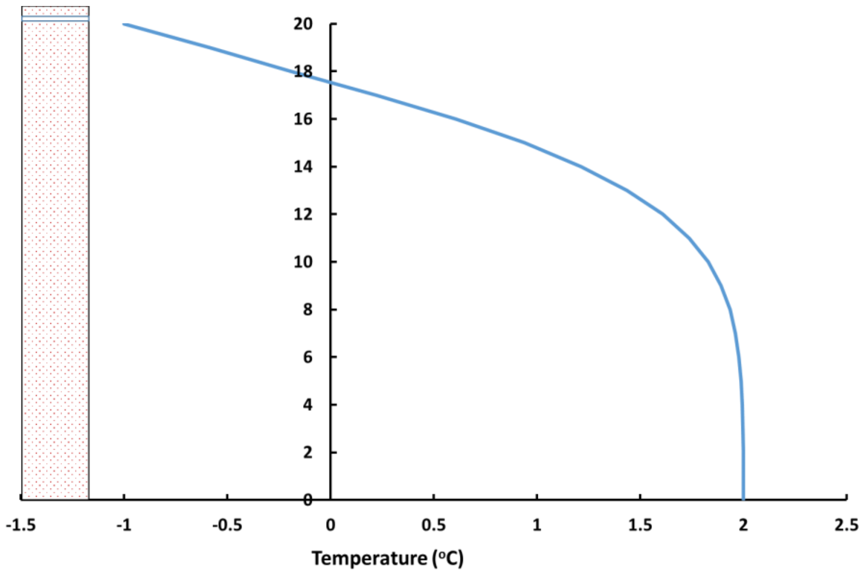




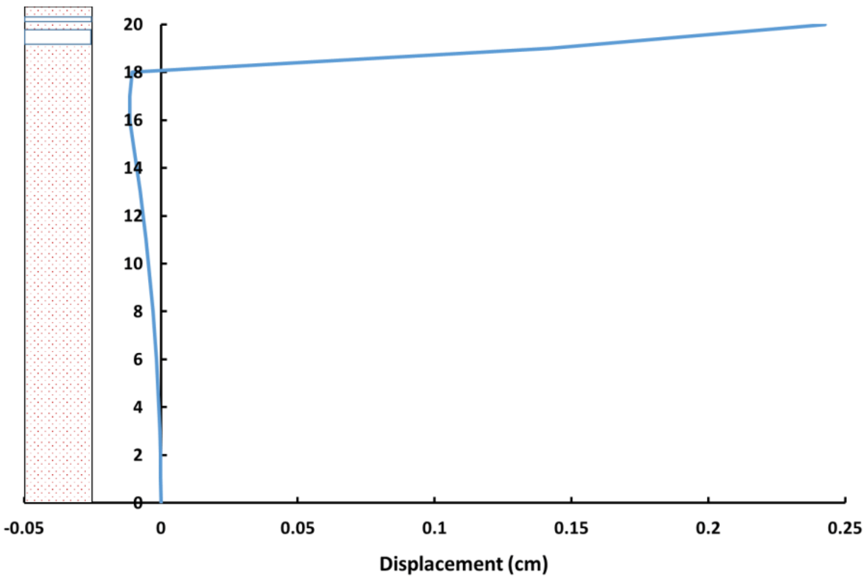
1-Dimensional Test (2nd ice lens)

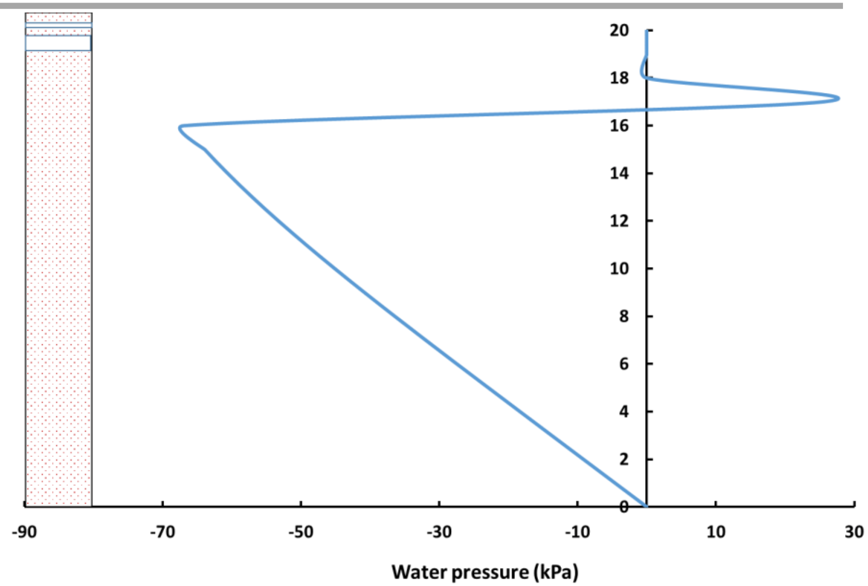


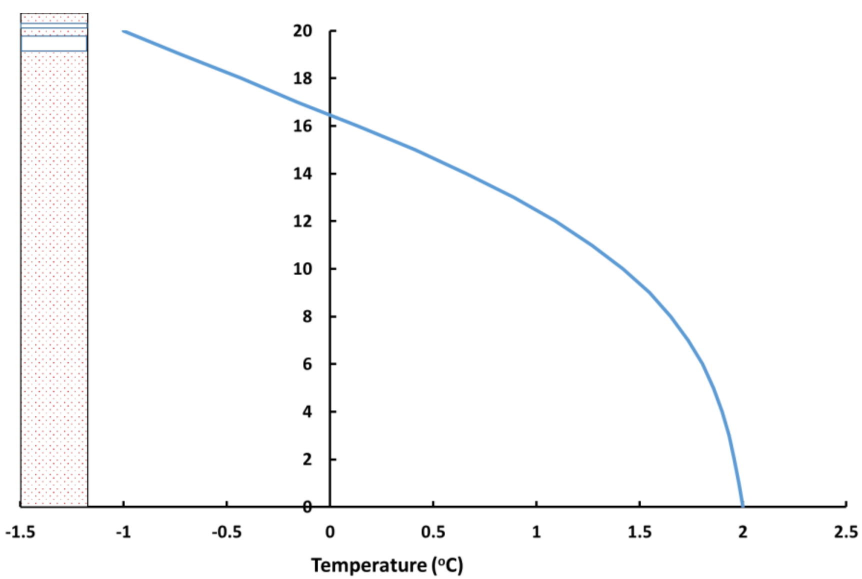




1-Dimensional Test (3rd ice lens)







Thanks for your attention!