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Energy Description for Immiscible Two-Fluid Flow in Porous Media by Integral Geometry and Thermodynamics

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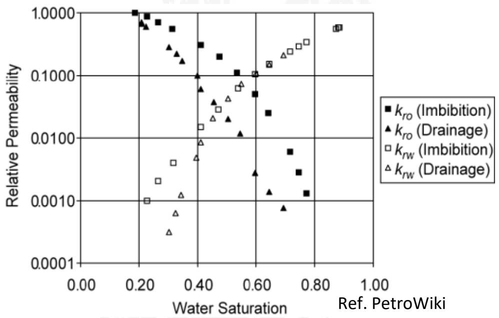
Two-fluid flow in porous media

- In a common approach, we use $k_{r\alpha}(S_\alpha)$ and macroscopic $P_c(S_w)$.

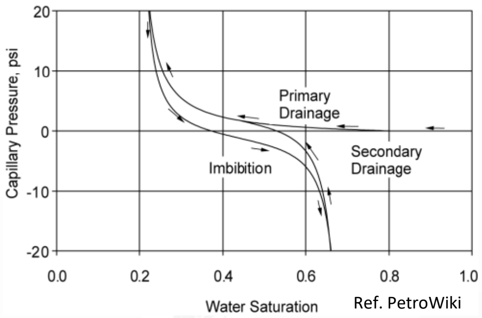
$$v_\alpha = -\frac{k_{r\alpha}(S_\alpha)k}{\mu_\alpha}\nabla p_\alpha, \quad \alpha = w, n$$

$$P_c(S_w) = P_n - P_w$$

- Pore scale phenomena affect the k_r and macroscopic flow.
- However, they are not well represented in the common approach (for ex. dependency of k_r on microscopic flow regimes).



Ref. PetroWiki

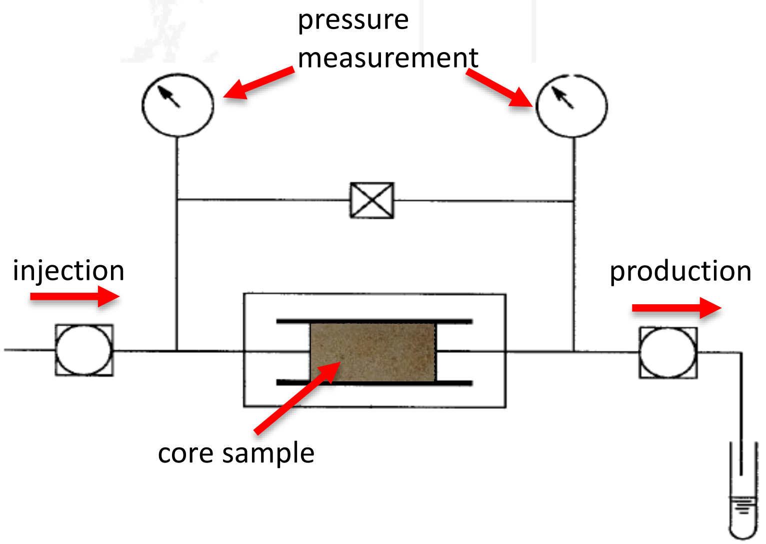


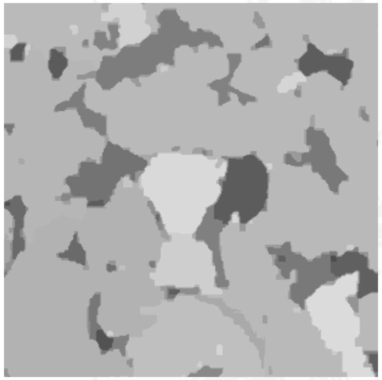
Ref. PetroWiki

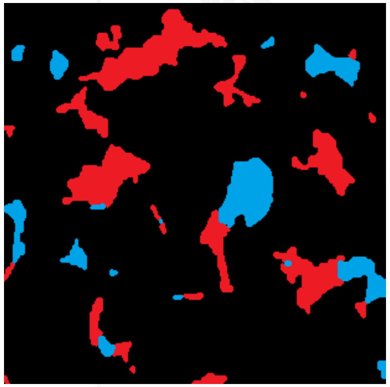
Darcy (1856), Wycoff & Botset (1936), Leverett (1941), Avraam and Payatakes (1995)

Recent advances in imaging

- From black-box coreflooding to fast X-ray micro-tomography





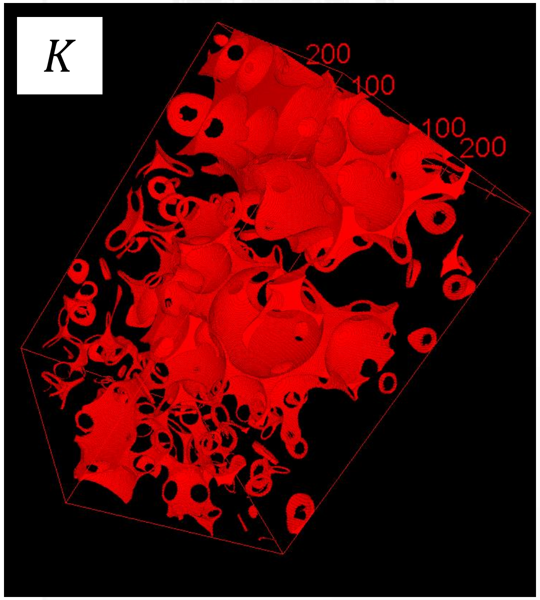


Intrinsic volumes in integral geometry

- For a 3D set of objects K, $V_0(\mathrm{K}) - V_3(\mathrm{K})$ denote the intrinsic volumes.

- Example of K: an assembly of fluid clusters

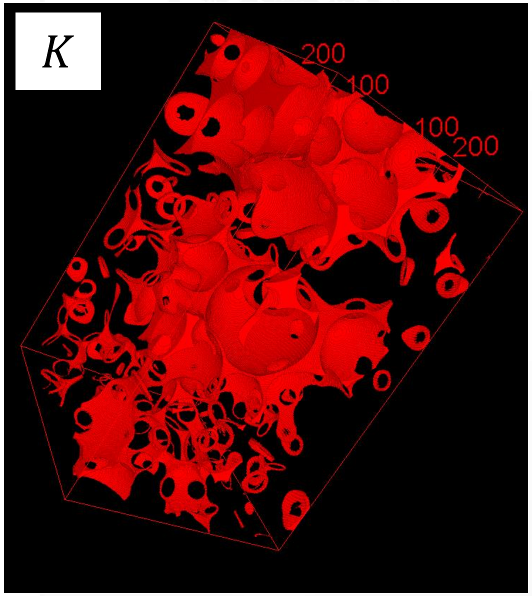
- $V_0(\mathrm{K}) = V(\mathrm{K})$
- $V_1(\mathrm{K}) = A(\mathrm{K}) = \int_{\delta K} dA$
- $V_2(\mathrm{K}) = H(\mathrm{K}) = \frac{1}{2}\int_{\delta K}(\kappa_1 + \kappa_2)dA$
- $V_3(\mathrm{K}) = \frac{1}{4\pi}\chi(\mathrm{K}) = \frac{1}{4\pi}\int_{\delta K}\kappa_1\kappa_2 dA$



Characterization theorem in integral geometry

- Under certain conditions, a function μ on K can be written as a linear combination of intrinsic volumes of K (Hadwiger 1975).

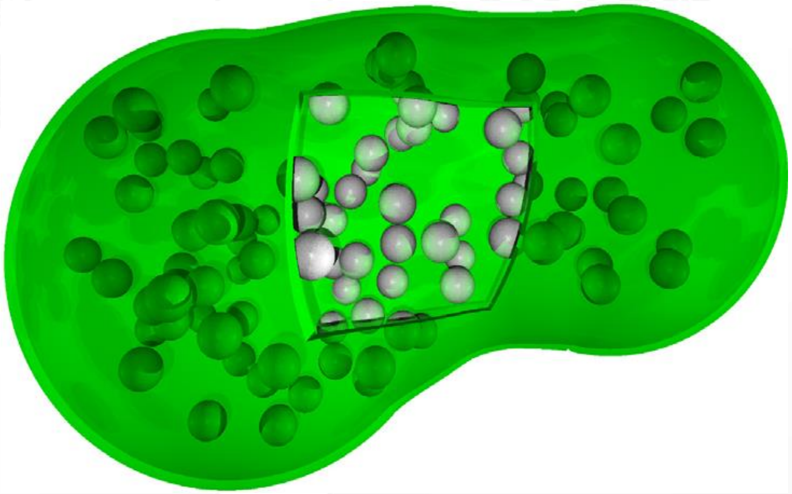
$$\mu(K) = \sum_{i=0}^{n} c_i V_i(K)$$



Free energy (*F*) of a fluid

- *F* of a fluid confined by a complex boundary fulfills the requirements of the Hadwiger theorem.
- Therefore, F can be described by a linear combination of intrinsic volumes (Mecke & Arns 2005).

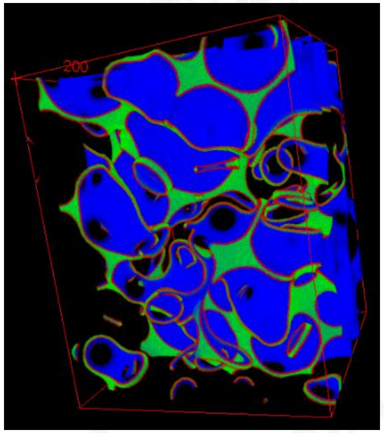
$$F(\mathrm{K}) = \sum_{i=0}^{n} c_i V_i(\mathrm{K})$$



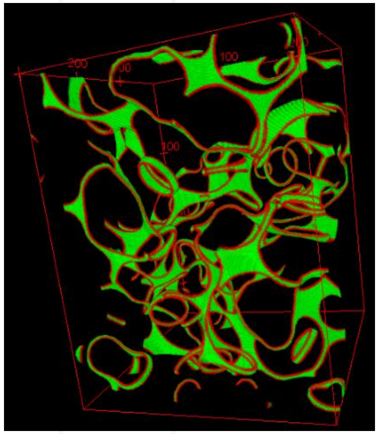
Mecke & Arns 2005

Free energy of a 2-fluid flow system (by integral geometry)

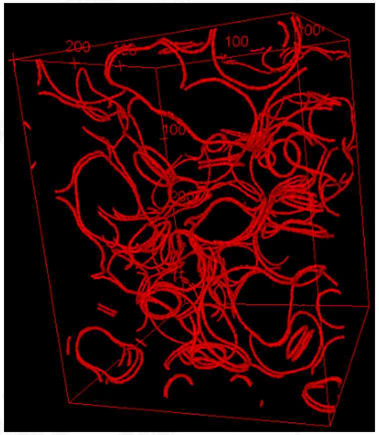
bulks (α, β, s)

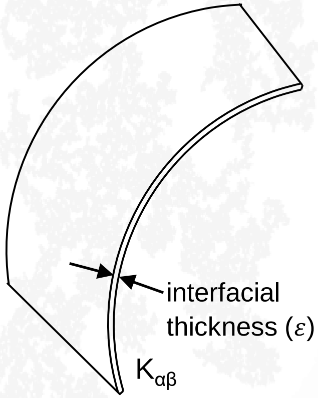


interfaces (αβ, αs, βs)



contact lines (αβs)





Interfaces and lines have volume.

$$F = F_s + F_\alpha + F_\beta - F_{\alpha\beta} - F_{\alpha s} - F_{\beta s} + F_{\alpha\beta s}$$

Dependencies of intrinsic volumes

$$F = F_s + F_\alpha + F_\beta - F_{\alpha\beta} - F_{\alpha s} - F_{\beta s} + F_{\alpha\beta s}$$

$$\begin{aligned} F &= F_s + \sum_{i=0}^{3} c_{i\alpha} V_{i\alpha} + \sum_{i=0}^{3} c_{i\beta} V_{i\beta} \\ &- \sum_{i=0}^{3} c_{i\alpha\beta} V_{i\alpha\beta} - \sum_{i=0}^{3} c_{i\alpha s} V_{i\alpha s} \\ &- \sum_{i=0}^{3} c_{i\beta s} V_{i\beta s} + \sum_{i=0}^{3} c_{i\alpha\beta s} V_{i\alpha\beta s} \end{aligned}$$

set	intrinsic volumes
$\boldsymbol{K_\alpha}$	V_α, A_α, H_α and χ_α
$\boldsymbol{K_\beta}$	V_β, A_β, H_β and χ_β
$\boldsymbol{K_s}$	V_s, A_s, H_s and χ_s
$\boldsymbol{K_{\alpha\beta}}$	$V_{\alpha\beta}$, $A_{\alpha\beta}$, $H_{\alpha\beta}$ and $\chi_{\alpha\beta}$
$\boldsymbol{K_{\alpha s}}$	$V_{\alpha\beta}$, $A_{\alpha s}$, $H_{\alpha s}$ and $\chi_{\alpha s}$
$\boldsymbol{K_{\beta s}}$	$V_{\beta s}$, $A_{\beta s}$, $H_{\beta s}$ and $\chi_{\beta s}$
$\boldsymbol{K_{\alpha\beta s}}$	$V_{\alpha\beta s}$, $A_{\alpha\beta s}$, $H_{\alpha\beta s}$ and $\chi_{\alpha\beta s}$

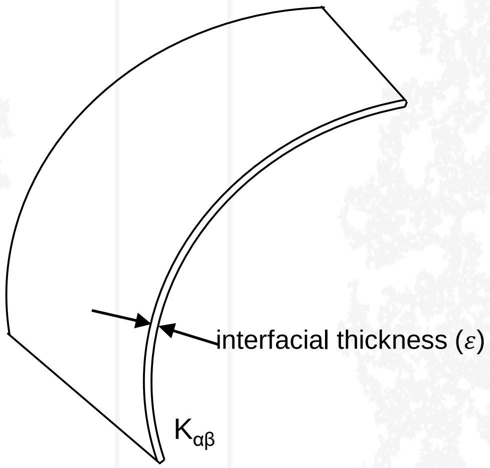
Dependencies of intrinsic volumes

- Simplifications (area)
- Only 2 of $(A_\alpha, A_\beta, A_{\alpha s}, A_{\beta s}, A_{\alpha\beta})$ are independent.

$$\begin{cases} A_\alpha - A_\beta + A_s = A_{\alpha s} \\ A_\beta - A_\alpha + A_s = A_{\beta s} \\ A_\alpha + A_\beta - A_s = A_{\alpha\beta} \end{cases}$$

Dependencies of intrinsic volumes

Simplifications (interfaces)



$$V_{0\alpha\beta} = \mathcal{O}(\varepsilon),\ \ \varepsilon \to 0$$

$$V_{1\alpha\beta} = A_{\alpha\beta}$$

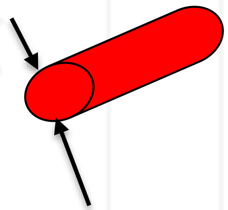
$$\boldsymbol{V_{2\alpha\beta}} = V_{2\alpha\beta,left} + V_{2\alpha\beta,right} + V_{2\alpha\beta,line} = +\frac{1}{4\pi^2}\int_{\delta K_{\alpha\beta s}} \left(\frac{1}{R_1} + \frac{1}{R_{2\to\infty}}\right) dA = \frac{1}{4\pi^2}\int_{\delta K_{\alpha\beta s}} \frac{dA}{R_1} =$$

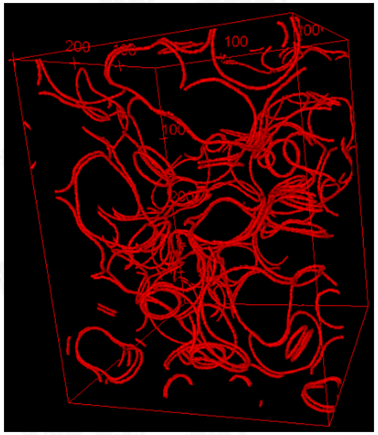
$$\frac{1}{4\pi^2}\int_{L_{\alpha\beta s}} \frac{\pi\frac{\varepsilon}{2}dL}{\frac{\varepsilon}{2}} = \boldsymbol{\frac{1}{4\pi} L_{\alpha\beta s}}$$

$$V_{3\alpha\beta} = \chi_{\alpha\beta}$$

Dependencies of intrinsic volumes

Simplifications (contact lines)





$$V_{0\alpha\beta s} = \mathcal{O}(\varepsilon^2),\ \varepsilon \to 0$$

$$V_{1\alpha\beta s} = \mathcal{O}(\varepsilon),\ \varepsilon \to 0$$

$$V_{2\alpha\beta s} = \frac{1}{4\pi^2}\int_{\delta K_{\alpha\beta s}} \left(\frac{1}{R_1} + \frac{1}{R_2 \to \infty}\right) dA = \frac{1}{4\pi^2}\int_{\delta K_{\alpha\beta s}} \frac{dA}{R_1} = \frac{1}{4\pi^2}\int_{L_{\alpha\beta s}} \frac{2\pi\frac{\varepsilon}{2}dL}{\frac{\varepsilon}{2}} = \frac{1}{2\pi}\boldsymbol{L_{\alpha\beta s}}$$

$$V_{3\alpha\beta s} = \int_{\delta K_{\alpha\beta s}} \frac{1}{(R_1)(R_2 \to \infty)} dA = 0$$

Dependencies of intrinsic volumes

- F is a function of 7 intrinsic volumes:

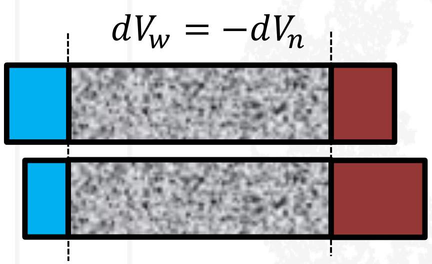
$$\hat{F} = \hat{F}_0 + \hat{c}_0 S_\alpha + \hat{c}_{1\alpha}\hat{A}_\alpha + \hat{c}_{2\alpha}\widehat{H}_\alpha + \hat{c}_{3\alpha}\hat{\chi}_\alpha + \hat{c}_{1\beta}\hat{A}_\beta + \hat{c}_{3\beta}\hat{\chi}_\beta + \hat{c}_{3\alpha\beta s}\hat{L}_{\alpha\beta s}$$

- Under extreme wetting conditions:

$$\hat{F} = \bar{F}_0 + \hat{c}_0 S_n + \bar{c}_{1n}\hat{A}_n + \hat{c}_{2n}\widehat{H}_n + \bar{c}_{3n}\hat{\chi}_n$$

Free energy (*F*) of a 2-fluid flow system (by thermodynamics)

$$dW_{external} = dF + dE_{dissipated}$$



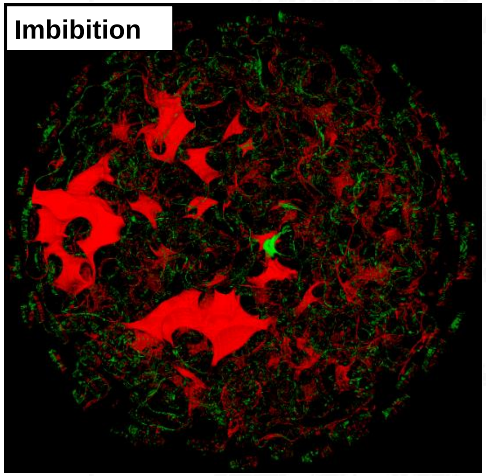
$$dW_{external} = P_w dV_w + P_n dV_n = (P_w - P_n) dV_n = P_c dV_n = \phi V P_c dS_n$$

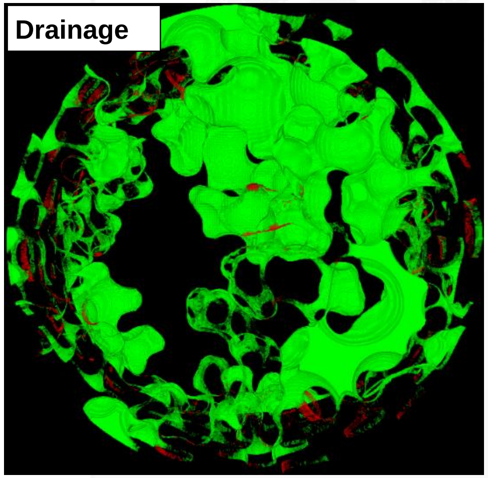
$$\Delta F = \phi V \int P_c dS_n + E_{dissipated}$$

Efficiency (E)

Imbibition $$E_I = -\frac{dW}{dF} = \phi V P_c \frac{dS_w}{dF}$$

Drainage $$E_D = \frac{dF}{dW} = -\frac{1}{\phi V P_c}\frac{dF}{dS_w}$$

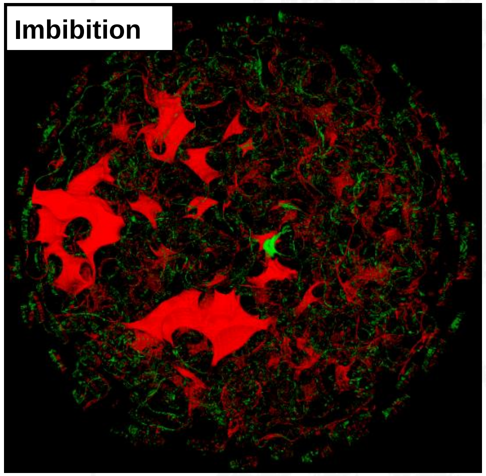


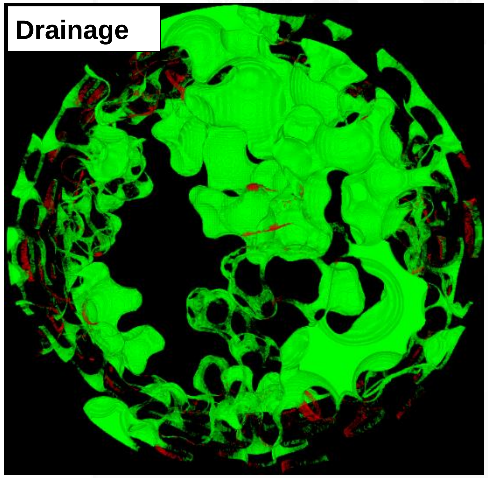


Efficiency (E)

- According to Morrow (1970), the imbibition had an efficiency of 92.5%, whereas the drainage had a lower efficiency.

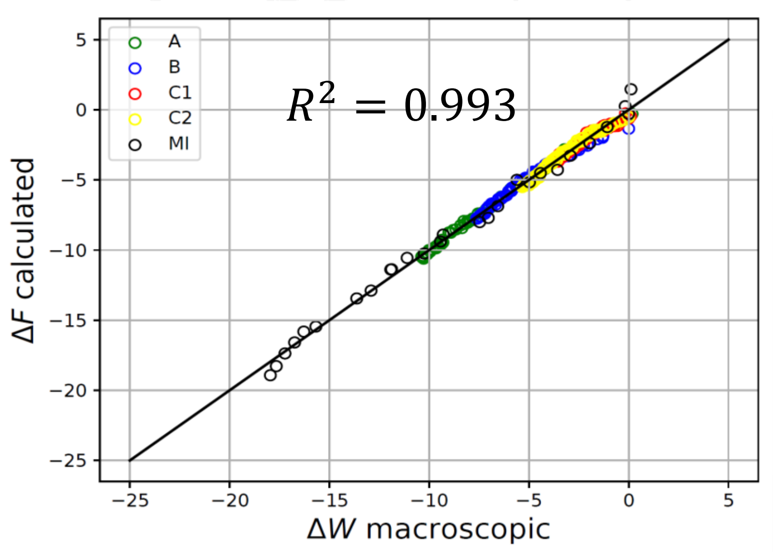
$$\Delta\hat{F} \simeq -0.925 \int P_c dS_w$$





Free energy (F) for spontaneous imbibition (SI)

- $\Delta\hat{F}(S_n) = 0.925 \int_{S_i}^{S_n} P_c dS_n$
- $\Delta\hat{F}(S_n) = \hat{c}_0 S_n + \hat{c}_{1n}\hat{A}_n + \hat{c}_{2n}\hat{H}_n + \hat{c}_{3n}\hat{\chi}_n + \hat{c}_{1w}\hat{A}_w + \hat{c}_{3w}\hat{\chi}_w + \hat{c}_{3nws}\hat{L}_{wns} - \hat{F}_i$

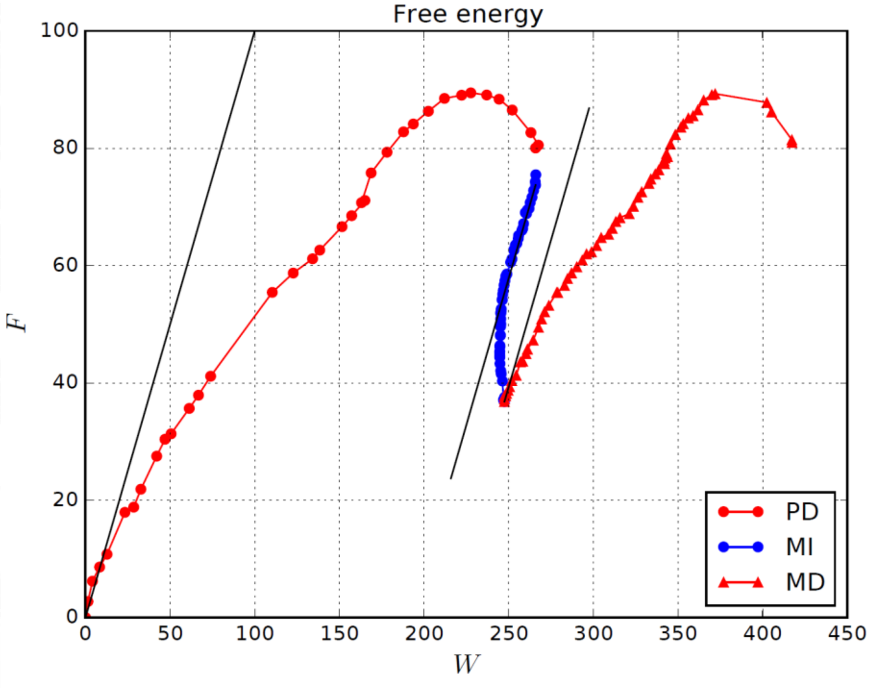


- Linear regression of 330 data points for five SI with macroscopic capillary number of 10^{-10} showed a good fit.
- Efficiency of 1 had similar results.

Efficiency for a PD-MI-MD experiment

$$\begin{cases} W = \int_{S_i}^{S_n} P_c dS_n \\ \hat{F}(S_n) = \hat{c}_0 S_n + \hat{c}_{1n}\hat{A}_n + \hat{c}_{2n}\hat{H}_n + \hat{c}_{3n}\hat{\chi}_n + \hat{c}_{1w}\hat{A}_w + \hat{c}_{3w}\hat{\chi}_w + \hat{c}_{3nws}\hat{L}_{wns} - \hat{F}_i \end{cases}$$

- 1-1 lines represent no dissipation.
- Early PD and MD have less dissipation (fewer irreversible processes, e.g. Haines jumps).
- Late PD and MD show a decrease in F when external work is applied.



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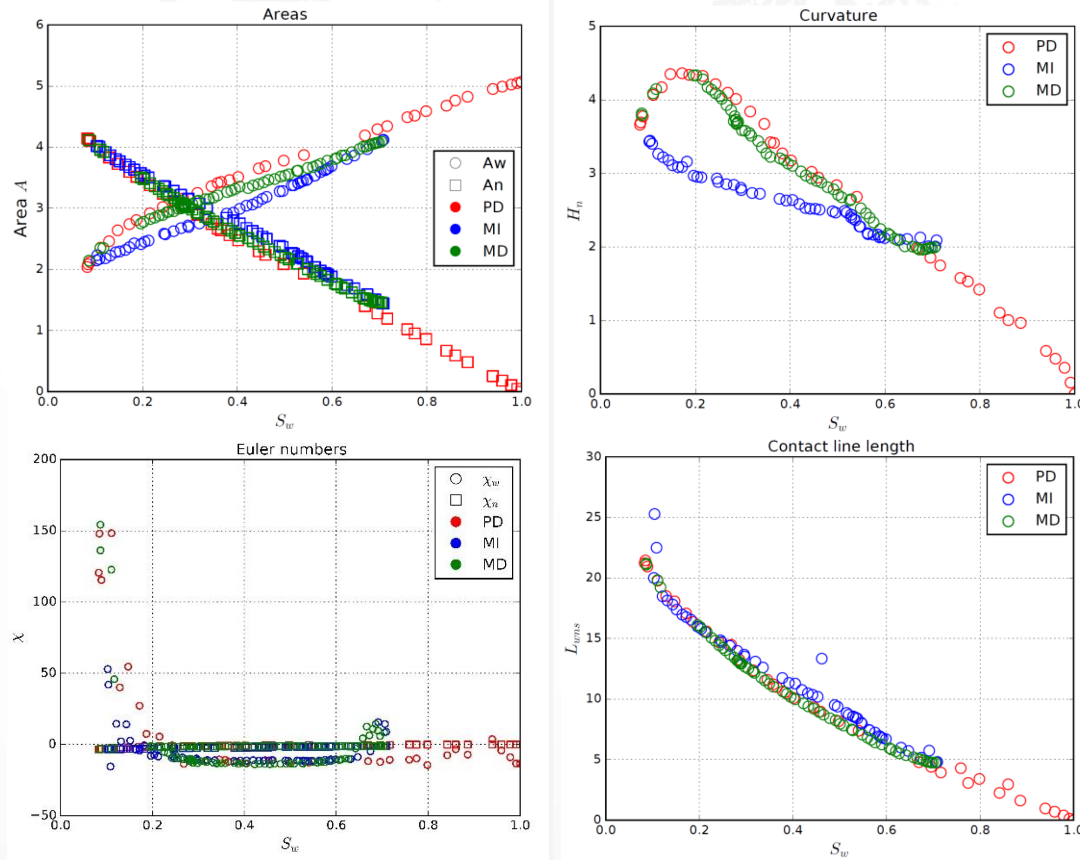
Work in progress…

- The coefficients from regression were correlated:
  - Limited temporal resolution or limited number of data points might not cover the possible nonlinear behavior.
- Possible non-geometric dependencies are not identified.

PD-MI-MD experiment



Conclusions

- Free energy is a function of 7 geometrically independent variables ($S_n, \hat{A}_n, \hat{A}_w, \widehat{H}_n, \hat{\chi}_n, \hat{\chi}_w, \hat{L}_{wns}$).
- The independent variables in extreme wetting conditions are ($S_n, \hat{A}_n, \widehat{H}_n, \hat{\chi}_n$).
- Dissipation of the processes can be calculated by combining integral geometry and thermodynamics.
- Possible non-geometric dependencies have yet to be found by further experimental and theoretical work.

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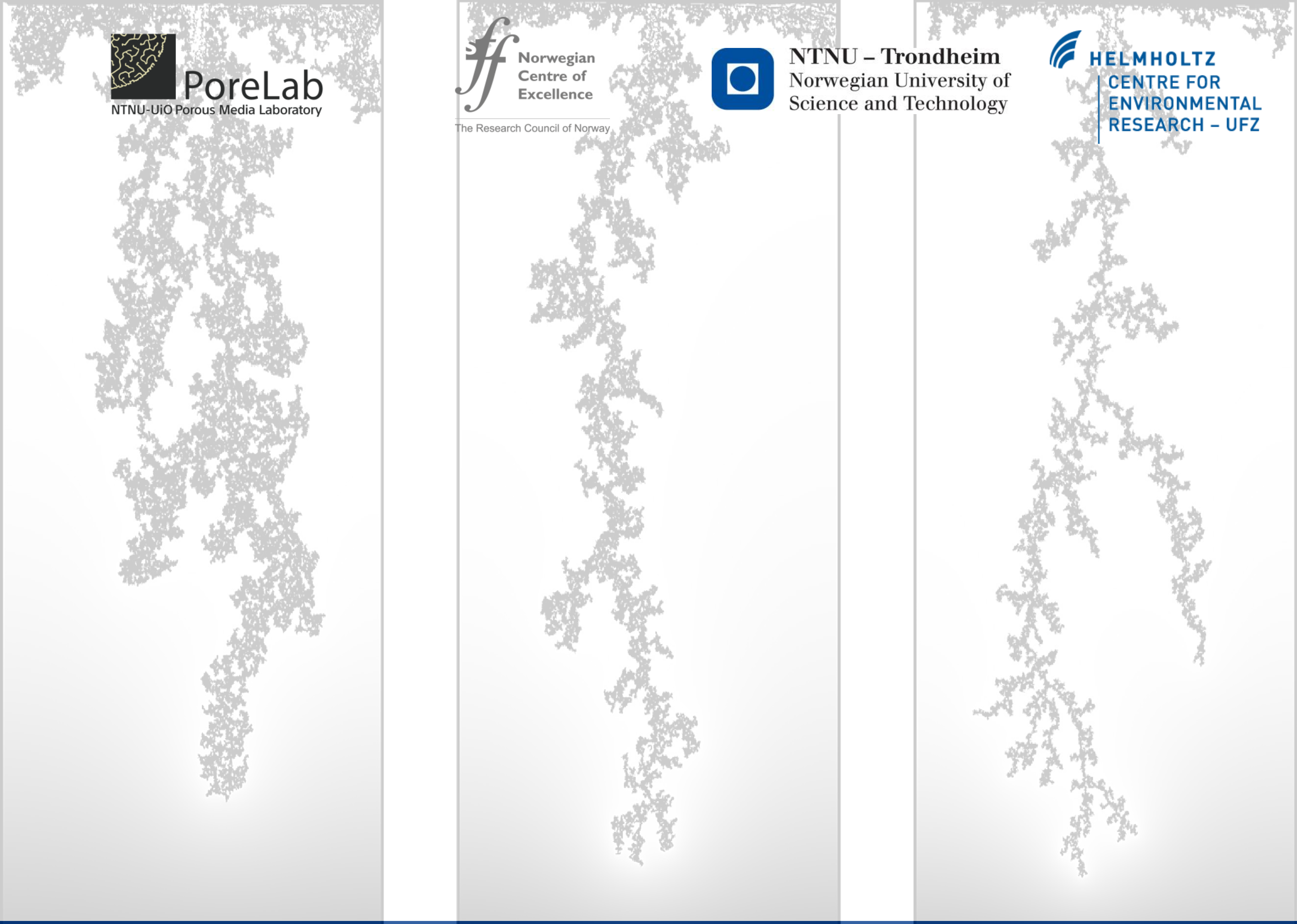
Thank You



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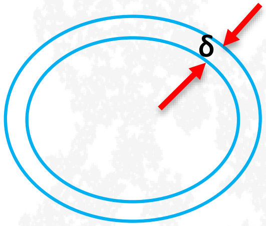
Integral geometry and 2-fluid flow

- Steiner formula is a relation between volume and other intrinsic volumes when the structure changes.

$$\Delta\epsilon^n = \frac{\lambda(\Omega_n \oplus \varsigma_\delta) - \lambda(\Omega_n)}{V} = \sum_{i=1}^{3} \frac{a_i M_i^n \delta^i}{V}$$



- Based on simulated non-wetting fluid configurations
  - $H_{wn}(S_n, A_{wn}, \chi_n)$ is valid as a state function.
  - $H_{wn}(S_n)$, $H_{wn}(S_n, A_{wn})$ are non-unique.

McClure et al. 2018

References*

- Darcy, H. (1856). Les Fontaines Publiques de la Ville de Dijon. Paris: Dalmont.
- Wyckoff, R. D., & Botset, H. G. (1936). The flow of gas-liquid mixtures through unconsolidated sands. *Journal of Applied Physics*, 7, 325.
- Leverett, M. C. (1941). Capillary behavior in porous solids. *Society of Petroleum Engineers*, 142(01), 152– 169. https://doi.org/10.2118/941152-G
- Avraam, D. G., & Payatakes, A. C. (1995). Flow regimes and relative permeabilities during steady-state two-phase flow in porous media. *Journal of Fluid Mechanics*, 293, 207– 236. https://doi.org/10.1017/S0022112095001698
- McClure, J. E., Armstrong, R. T., Berrill, M. A., Schlüter, S., Berg, S., Gray, W. G., & Miller, C. T. (2018). Geometric state function for two-fluid flow in porous media. *Physical Review Fluids*, 3, 084306. https://doi.org/10.1103/PhysRevFluids.3.084306
- Hadwiger, H. (1957). Vorlesungen über Inhalt, Oberfläche und Isoperimetrie (Lecture on Content, Surface and Isoperimetry) (p. 312). Berlin-Heidelberg: Springer-Verlag. https://doi.org/10.1007/978-3-642-94702-5
- Morrow, N. R. (1970). Physics and thermodynamics of capillary action in porous media. *Industrial and Engineering Chemistry*, 62(6), 32– 56. https://doi.org/10.1021/ie50726a006
- Mecke, K., & Arns, C. H. (2005). Fluids in porous media: A morphometric approach. *Journal of Physics. Condensed Matter*, 17(9), S503– S534. https://doi.org/10.1088/0953-8984/17/9/014

* Check the fulltext article at https://doi.org/10.1029/2018WR023619 for details of this work and a complete list of references.

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