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Norwegian Centre of Excellence
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NTNU
Norwegian University of Science and Technology

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Time Correlation Functions of Immiscible Two-Phase Flow in Porous Media

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SK70, 29.-30. August
Trondheim

Steady state flow in a 2D Hele-Shaw Cell

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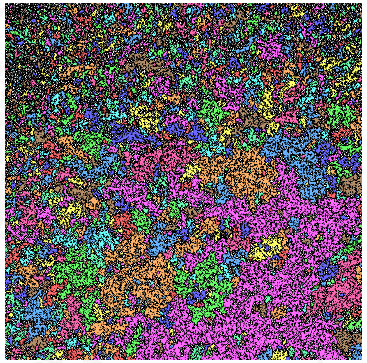
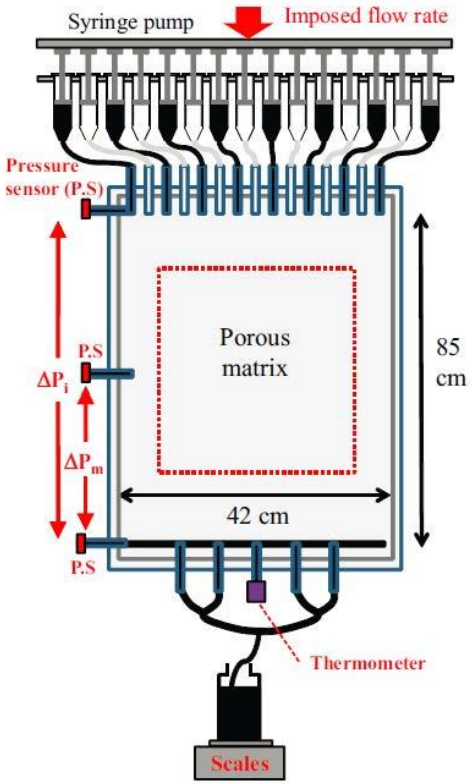
The macroscopic system parameters remain constant – or change slowly on the scale of the fluctuations.



Both fluids move and fluid clusters break up and merge; still steady state.

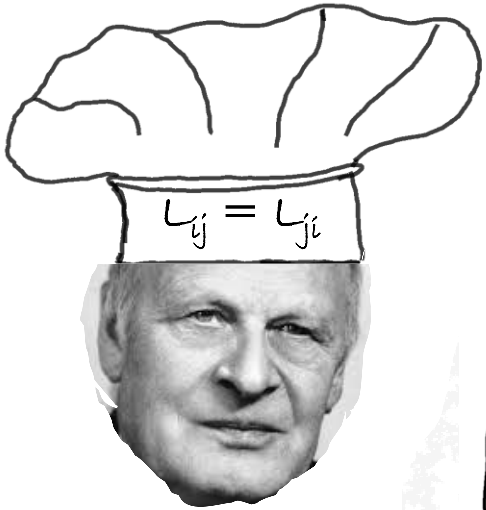


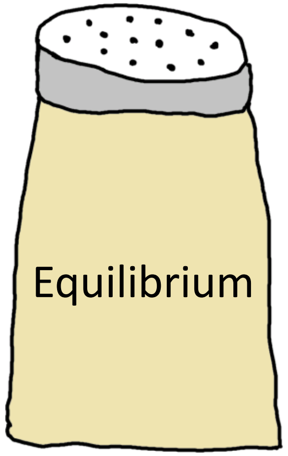
- Tallakstad et al., Phys. Rev. Lett. **102**, 074502 (2009); Phys. Rev. E **80**, 036308 (2009).
- Erpelding et al. Phys. Rev. E **88**, 053004 (2013).
- Aursjø et al. Frontiers in Physics. **2**, 63 (2014).

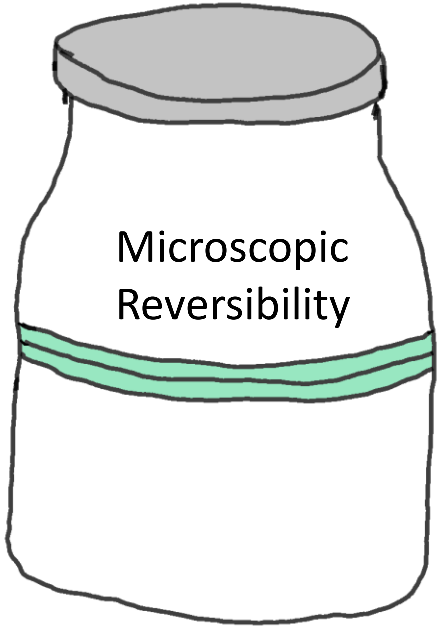
Cooking with Onsager

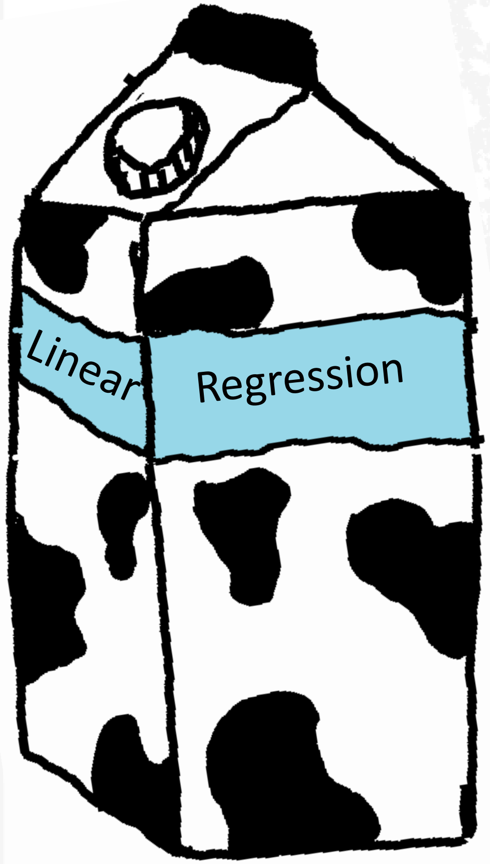
Reciprocal Relations:

(For **Non**-Thermal Fluctuations !)









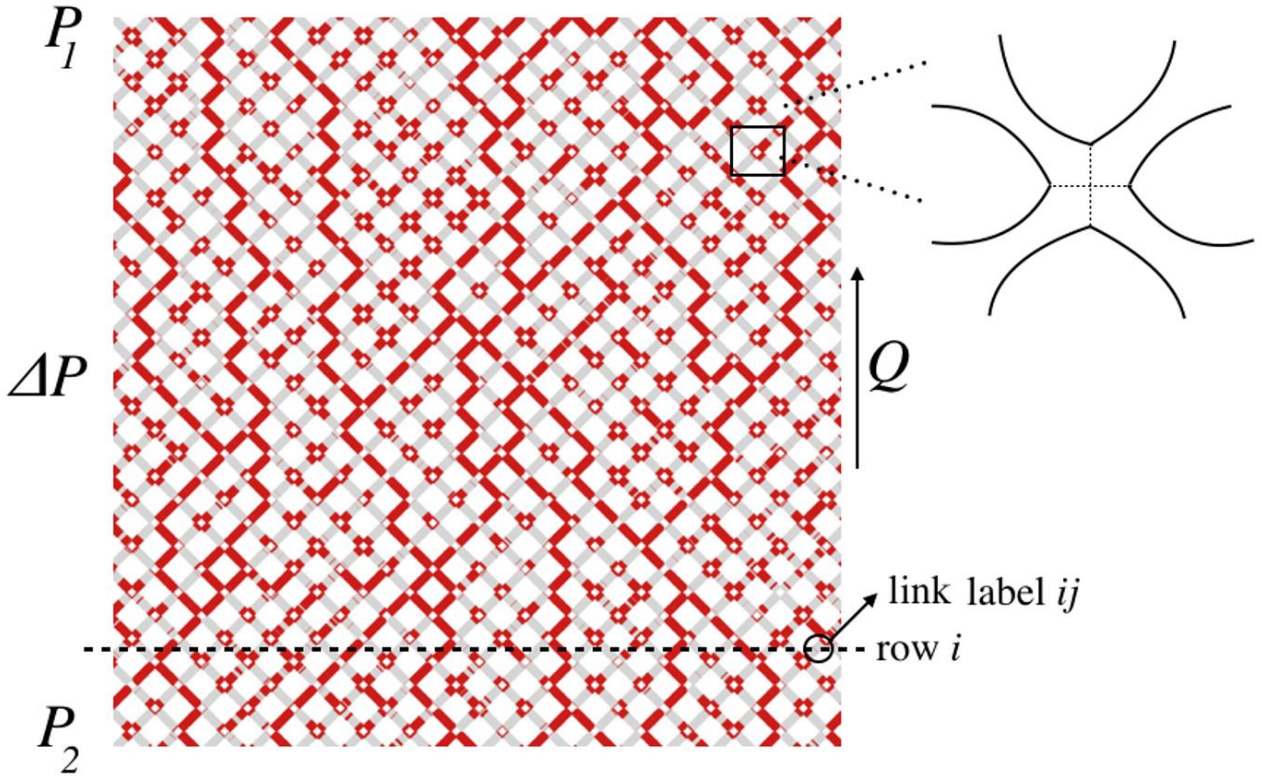
$$\mathbf{x} = \{x_1, x_2, ..., x_M\}$$

$$\wp(\mathbf{x}) = \frac{1}{W} \exp \frac{\mathbf{x}^T S \mathbf{x}}{2k}$$

$$\langle x_i(\tau) \mid x_j(0) \rangle = \langle x_j(\tau) \mid x_i(0) \rangle$$

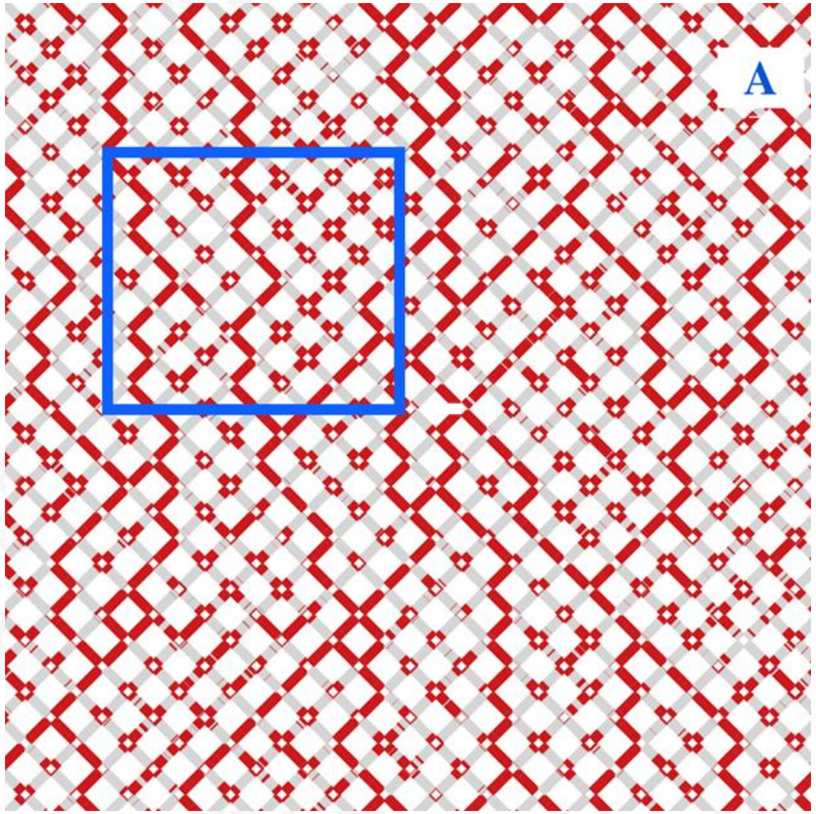
$$\frac{dx_i(t)}{dt} = -\sum_j L_{ij} x_j(t)$$

Network Model



- Distribution of pore radii
- Constant pressure drop across network
- Flow in each pore between node i and j given by : $q_{ij} = \frac{g_{ij}}{l_{ij}}\left(p_j - p_i - p_c\right)$

Testing the ergodic hypothesis



Steady state of total flow, given the saturation, was

I) Computed as time average of the total system
II) Computed as ensemble average over subvolume V

I. Time averaging procedure:

Solve Kirchhoff equations for the network, Integrate flow equation over time until the total flow is constant

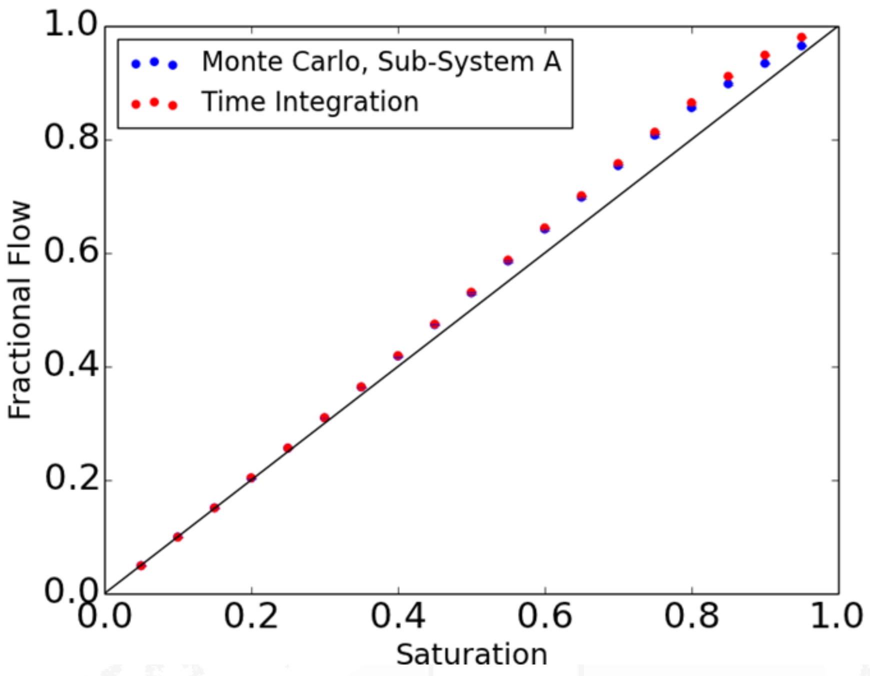
II. Metropolis Monte Carlo algorithm:

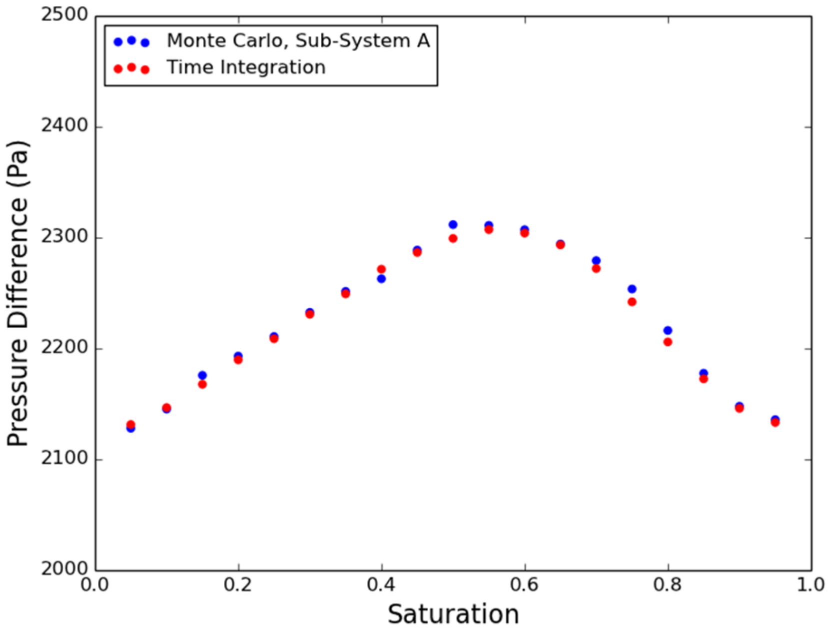
Take out a subvolume **V**. Run as in I. Return.
Reject/Accept according to Metropolis algorithm.

Savani et al. Transp. Porous Med. (2017), Phys. Rev. E (2017)

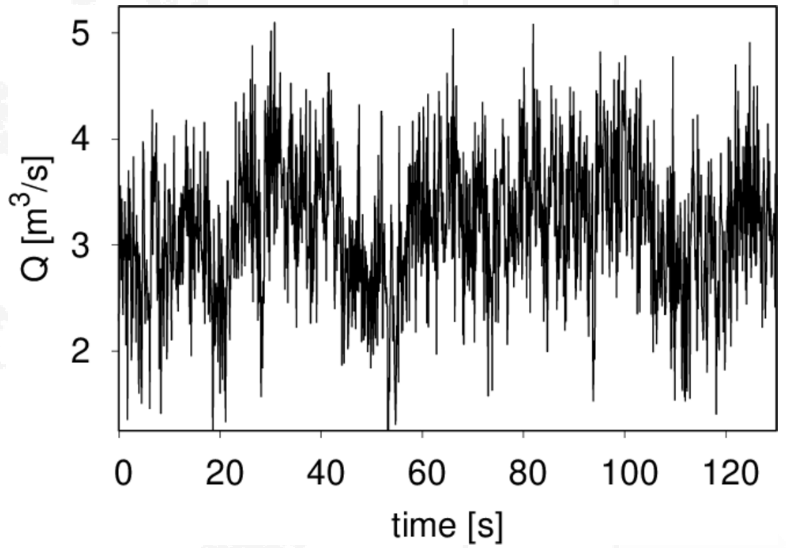
Ergodic hypothesis obeyed!

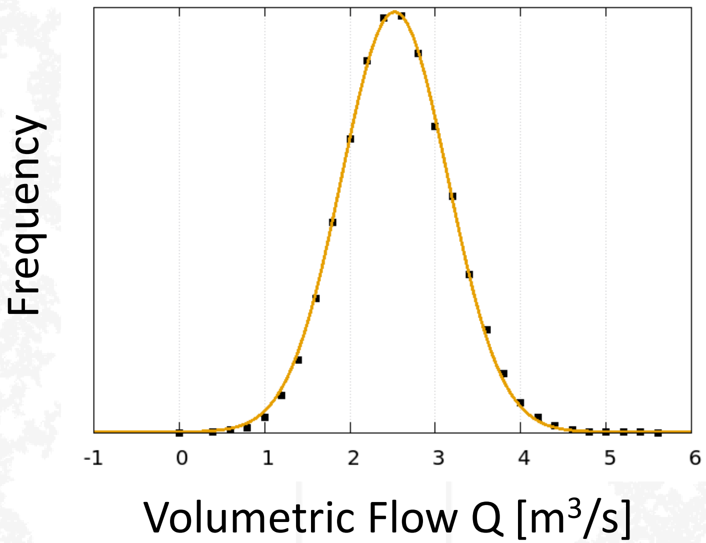
Ca = 0.1, Fluids with the same viscosity





Fluctuations





Size 30x20

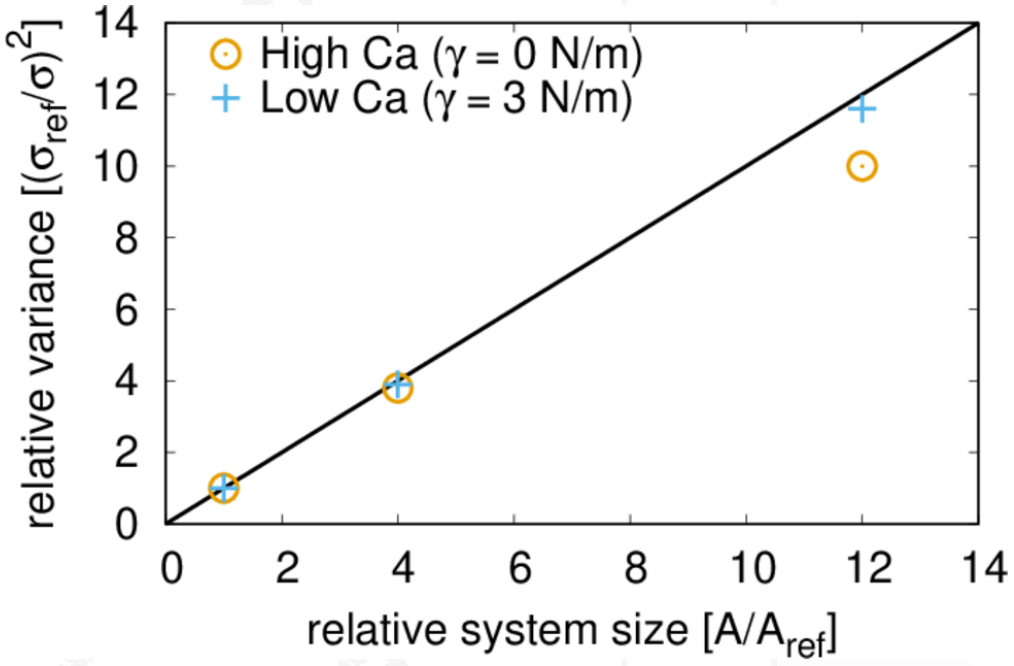
Ca = 10^{-3}-10^{-2}

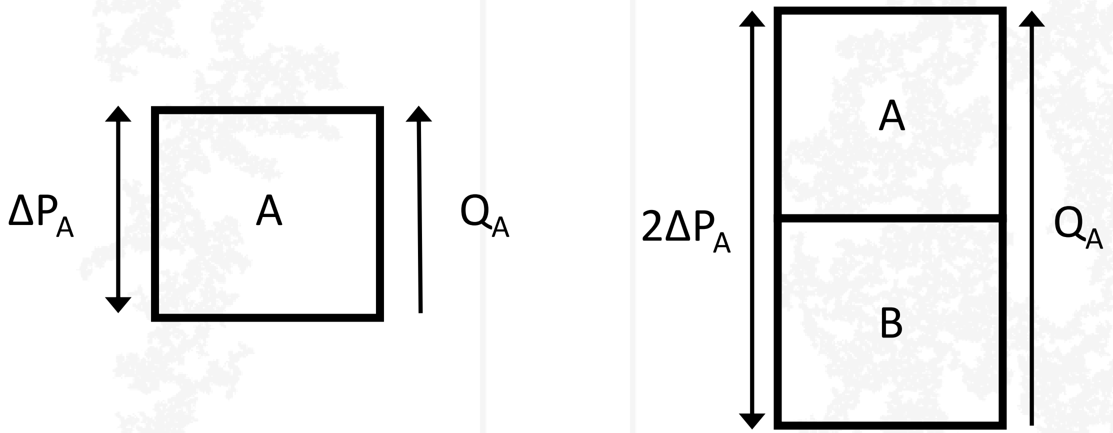
$\mu_w=\mu_{nw}$=0.01 Pa s

σ=3 N/m

Gaussian !

REV

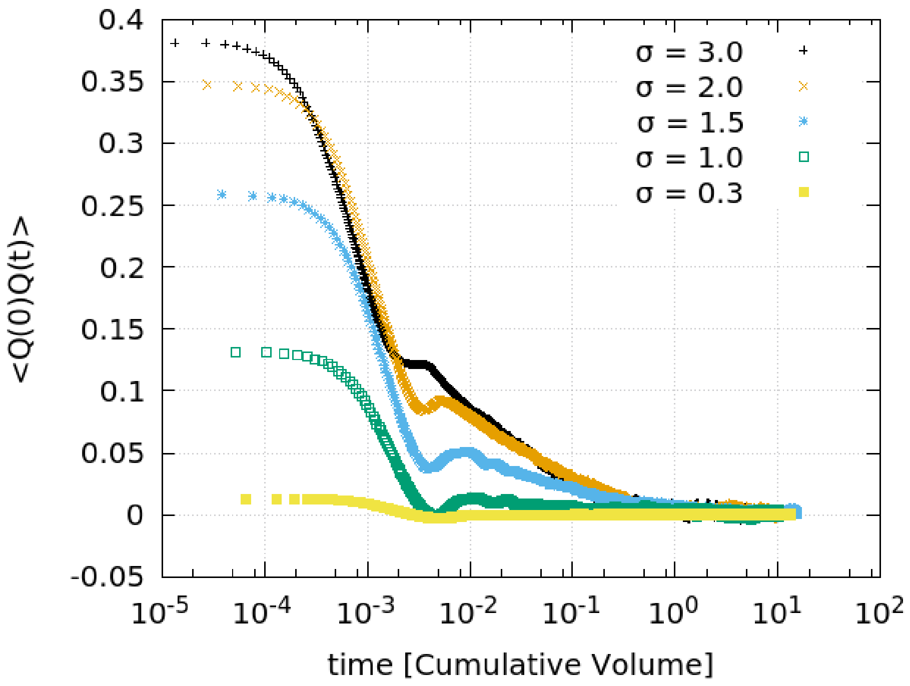


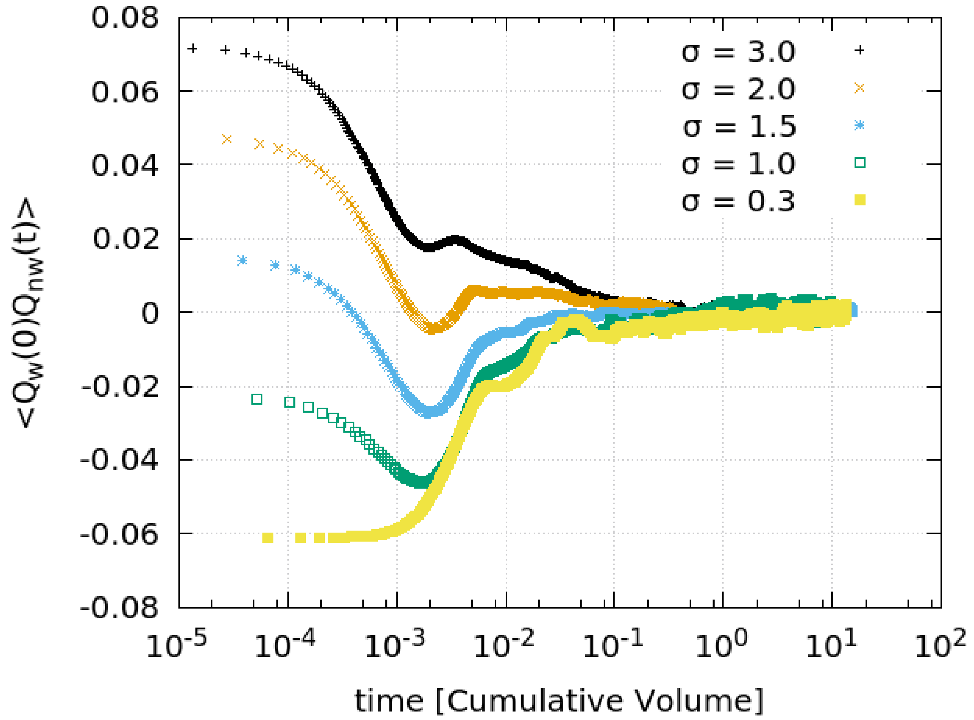


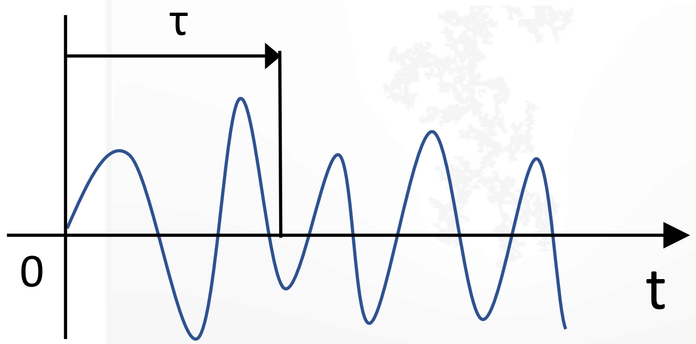
$$S_{AB} = S_A + S_B$$

Low Ca, Ca = 10^{-3}-10^{-2}
High Ca, Ca = inf

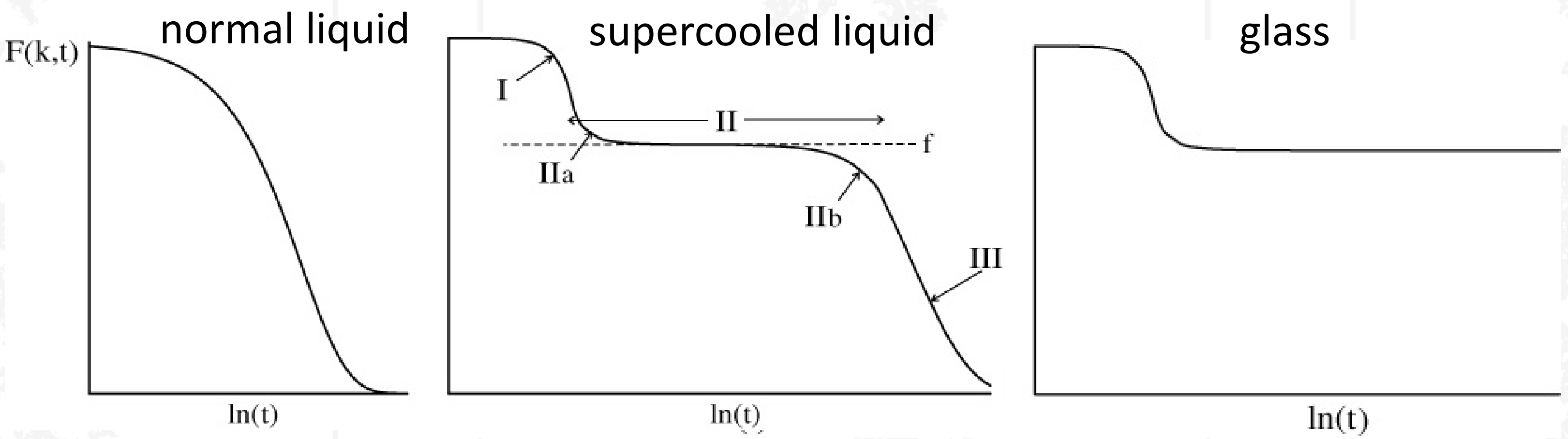
Time Correlation Function

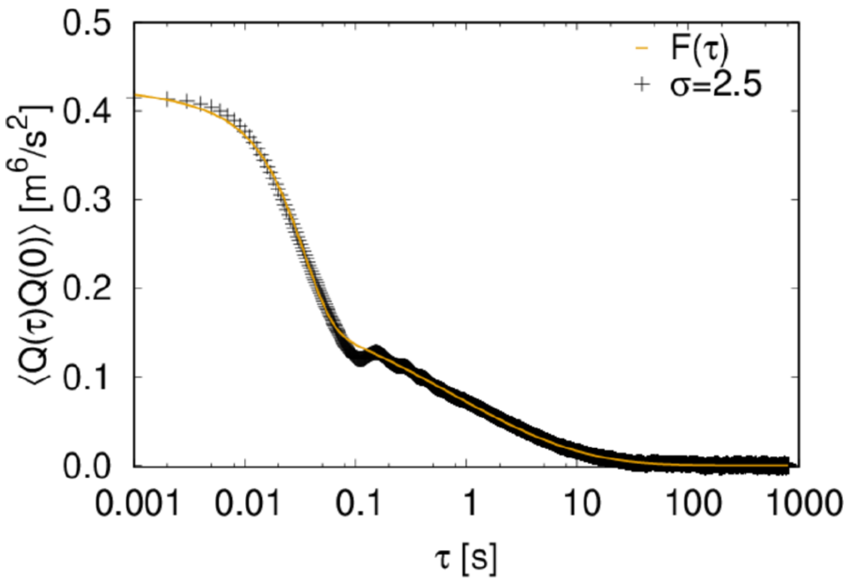






Similarities to Glassy Systems





$$F(t) = a\exp[-(t/\tau_1)^{\alpha}] + b\exp[-(t/\tau_2)^{\beta}]$$

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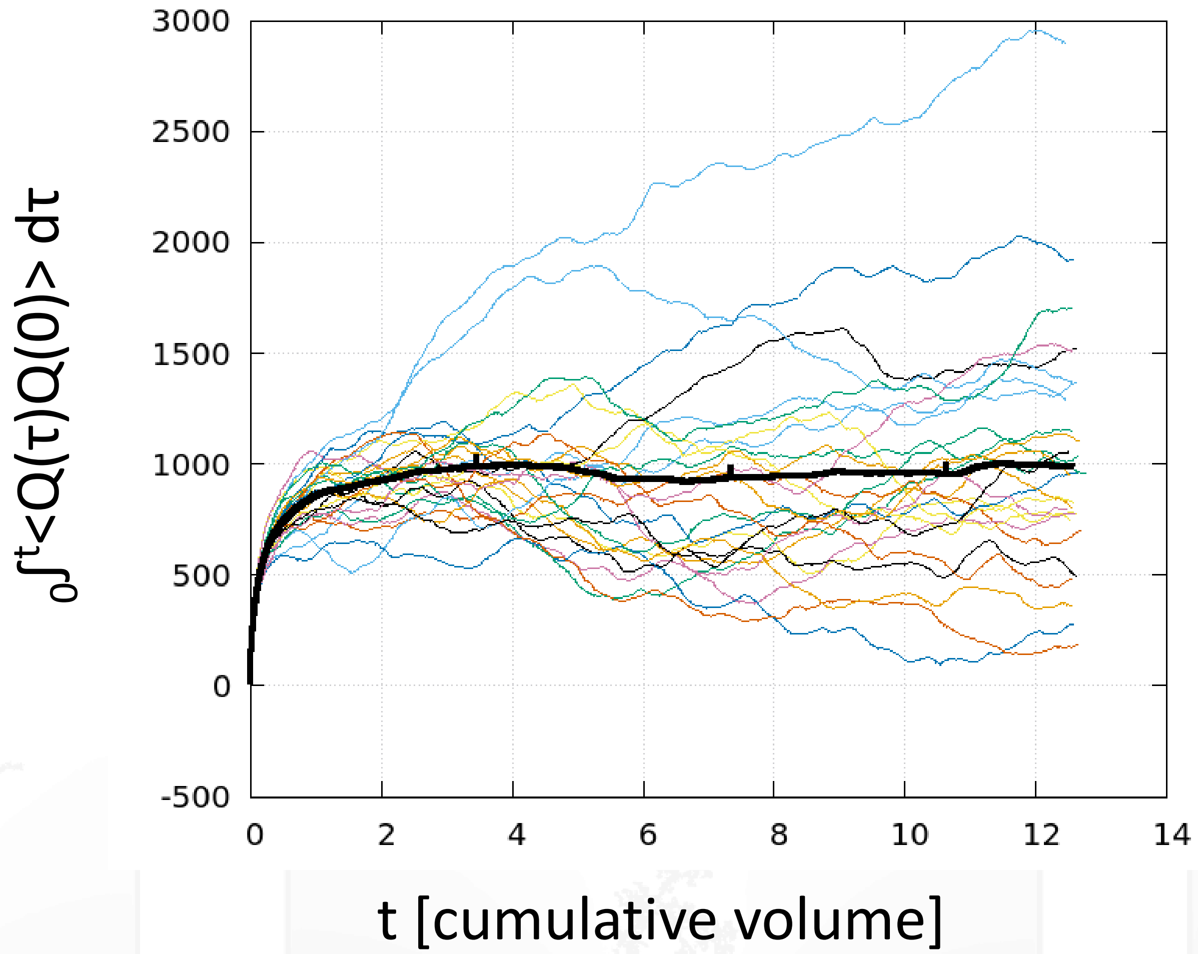
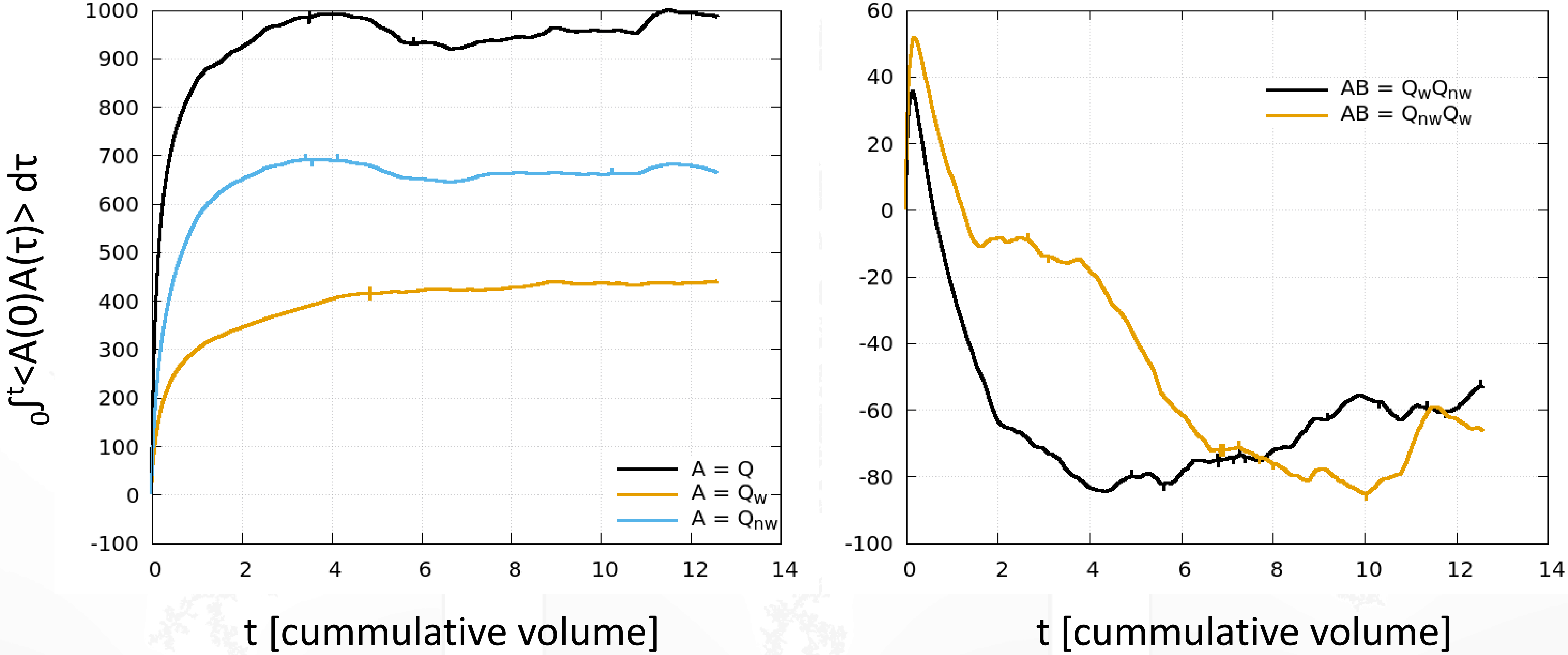
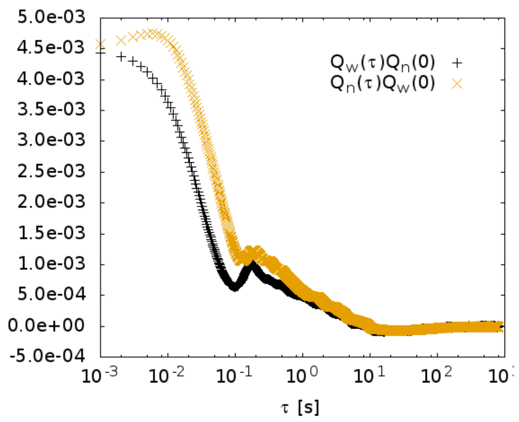
Convergence



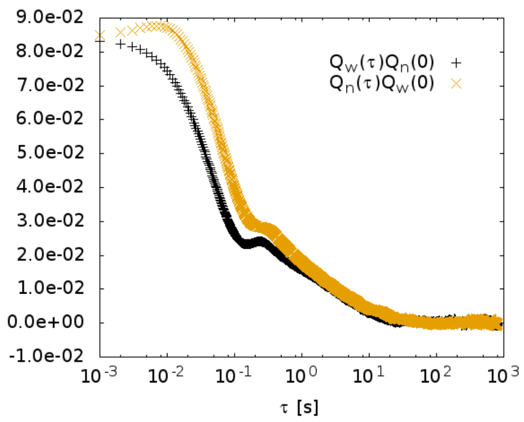

$_0\int^t$<A(0)A(τ)> dτ
1000
900
800
700
600
500
400
300
200
100
0
-100
0 2 4 6 8 10 12 14
A = Q
A = Q_w
A = Q_{nw}
t [cummulative volume]
60
40
20
0
-20
-40
-60
-80
-100
0 2 4 6 8 10 12 14
AB = Q_wQ_{nw}
AB = $Q_{nw}Q_w$
t [cummulative volume]

Low Ca

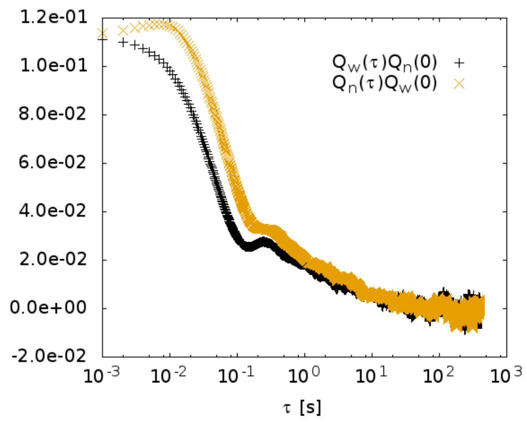
30x20



60x40

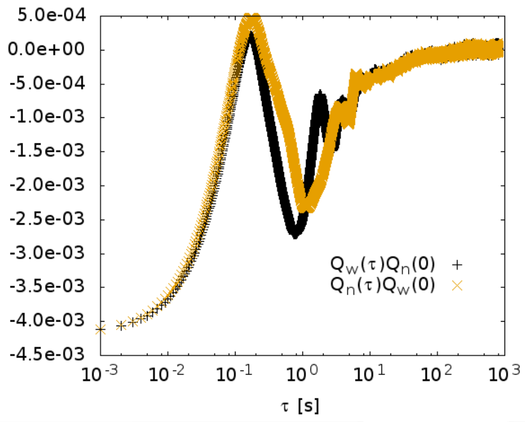


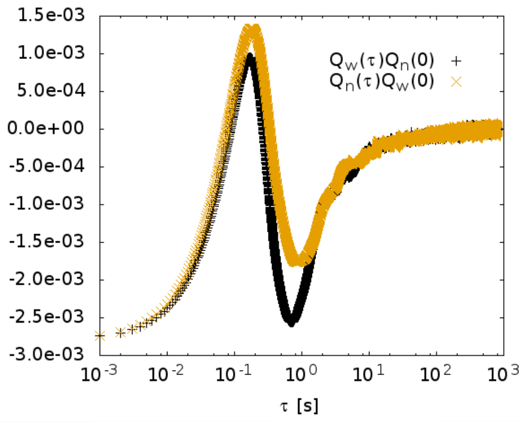
120x60

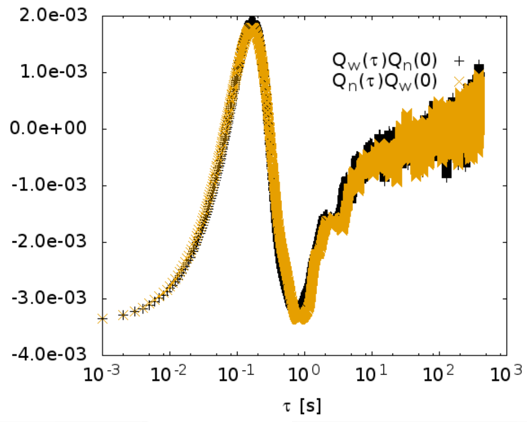


Increasing system size

High Ca







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Onsager Reciprocal Relations Apply

Low Ca, Sw = 0.5

$\Lambda_{ii} \cdot 10^3$ [m^6/s]	ΔP = 10 [N/m^2]	ΔP = 15	ΔP = 20
Λ_{ww}	456	722	1588
Λ_{nn}	690	1514	1879
Λ_{wn}	-106	-346	-729
Λ_{nw}	-99	-320	-703

High Ca, ΔP = 10 [N/m^2]

$L_{ii} \cdot 10^3$ [m^6/s]	S_w = 0.25	S_w = 0.5	S_w = 0.75
Λ_{ww}	1	5	9
Λ_{nn}	21	90	191
Λ_{wn}	-5	-19	-46
Λ_{nw}	-5	-22	-47

Transport Coefficients

Time correlation functions are related to transport coefficients:

e.g. Molecular diffusion:

$$D = \lim_{t \to \infty} \frac{1}{2t} \langle |\, x(t) - x(0) \,|^2 \rangle$$

$$x(t) = x(0) + \int_0^t dt' v(t')$$

$$|\, x(t) - x(0) \,|^2 = \int_0^t dt' v(t') \int_0^t dt'' v(t'')$$

$$D = \int_0^\infty d\tau \langle v(\tau) \cdot v(0) \rangle$$

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Transport Coefficients

Time correlation function of the average velocity or flow are related to collective diffusion and Maxwell-Stefan Diffusion coefficient (if more then one species present) :

$$\Lambda_{ik} \propto \int_0^\infty d\tau \langle J_i(\tau) \cdot J_k(0) \rangle$$

The Maxwell-Stefan diffusion coefficient for a two component system is proportional to:

$$D_{MS} \propto \frac{x_2}{x_1}\Lambda_{11} + \frac{x_1}{x_2}\Lambda_{22} - \Lambda_{12} - \Lambda_{21}$$

Diffusion and Friction coefficient

$$c_i \nabla \mu_i = \sum_{j \neq i} K_{ij}(v_j - v_i)$$

$$K_{ij} \propto \frac{1}{D_{MS,ij}}$$

$$\frac{v_j - v_i}{D_{MS,ij}}$$

ΔP = 10 [N/m^2]	ΔP = 15	ΔP = 20
0.24	0.23	0.24

Conclusions

- Auto and cross correlation functions of immiscible two-phase flow do converge
- Cross coefficients obey Onsager Reciprocal Relation within the given accuracy
- Transport coefficients may be estimated from steady state simulations
- The framework of Non-equilibrium «thermodynamics» applies to this system

Thank you for your Attention !