Out-of-equilibrium self-assembly for the formation of biological and soft-matter structures

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Congratulations Signe!!

$SKe^{4.25}$
Age is just a number!

Three type of ages:

-Official (ID)
-Biological (how do you feel)
-Relativistic (time passes slowly when activity is very intense)
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Outline

1. Out of equilibrium self-assembly

2. Gelation

3. Liesegang patterns

4. Determining the architecture of self-assembled structures from energy dissipation

\[ \eta_{\text{ph}} = \frac{l_{d}d}{\max(l_{u}, l_{v}, l_{o})^3} \]
Out-of-equilibrium self-assembly

- Self-assembly (SA) is the organization of discrete elements into structures.
- An assembled structure is an arrangement of building blocks (BB) into a material object that integrates a heterogeneous system.
- Equilibrium SA leads to the formation of stable structures through quasi-equilibrium steps.

Nonequilibrium self-assembly (NESA) leads to the formation of stable, metastable, kinetically-trapped and stationary structures mediated by dissipation.

NESA processes are the previous steps in the formation of self-organized (SO) structures.
Structures

A: Microtubules, assembled from tubuline dimers.
B: Gel, assembled from fibers or microtubules.
C: Plant cells, assembled from microtubules assemblies.
D: Leaf, assembled from plant cells
Self-propelled “particles”

Structures

Magneto-hydrodynamic assemblies

Microtubule-based structures, active gels

A. Networks (contractile or as cellular structure)
B. Microtubule asters
C. Microtubule vortex
Questions

• How the formation of the structures is governed by the laws of thermodynamics?

• What is the amount of energy dissipated in the formation of a non-equilibrium self-assembled structure from a set of disordered elements?

• In the lack of a non-equilibrium potential that can constitute a structure selection mechanism, as occurs in equilibrium, we wonder why under specified conditions a determined structure appears and not another.

• Computational approaches have technical challenges because the number of particles needed to explain the mesoscale behavior is very large.

• These methods enable the possibility of simulating larger systems for longer time, but they do not enable the quantification of the dissipation (entropy production) naturally.
Gel formation is a NESA process.

- Model based on mesoscopic nonequilibrium thermodynamics*.
- The models gives:
  i) fiber orientation probability
  ii) dissipation (entropy production).

**Mechanism**

**BB**: N,N-dibenzoyl-(L)-cystine (DBC)

**Activator**: Methyl Iodide (MeI)

**Activated blocks**: DBC−Me_2

**Intermediate activated BB**: DBC−Me

**First-order structures**: linear fibers formed from DBC by sequential reactions

**Second-order structures**: gel formed by agglomeration of linear fibers.

REACTION COORDINATES
A $\rightarrow$ B

\[ \mu_A, \mu_B \rightarrow \mu(\gamma) \]
\[ c_A, c_B \rightarrow c(\gamma) \]
\[ s \rightarrow s(\gamma) \]

Mesoscopic description:

\[ Tds(\gamma) = -\mu(\gamma)dP(\gamma) \]

Mesoscopic entropy production

\[ \sigma(\gamma) = -\frac{1}{T} J(\gamma) \frac{\partial \mu}{\partial \gamma} \]
\[ J(\gamma) = -\frac{L}{T} \frac{\partial \mu}{\partial \gamma} \]

J.M. Vilar, J.M. Rubi, PNAS (2001)

\[ \frac{\partial P}{\partial t} = \frac{\partial}{\partial \gamma} D \left( \frac{\partial P}{\partial \gamma} + \frac{P}{k_B T} \frac{\partial \Phi}{\partial \gamma} \right) \]
\[ \mu = k_B T \ln P + \Phi \]

**Local fugacity**

\[ z(\gamma) = e^{\frac{\mu(\gamma)}{k_B T}} \]

\[ J = -k_B L \frac{\partial z}{\partial \gamma} \]

\[ D = k_B \frac{L}{z} \]

\[ \bar{J}(t) = \int_{1}^{2} J d\gamma = -D(z_2 - z_1) = -D(e^{\mu_2} - e^{\mu_1}) \]
MODEL

Phase Space: \[ \Gamma_n = (\gamma_n, \gamma'_n, \lambda_n, \theta_n) \]

Continuity: \[ \frac{\partial p_n(\Gamma_n, t)}{\partial t} = -\nabla_{\Gamma_n} \cdot J_n(\Gamma_n, t) \]

Entropy production: (1st order)
\[ \sigma_n = -\frac{1}{T} \sum_{i=1}^{4} \int_{\Gamma_n^{(i)}} J_n^{(i)} \frac{\partial \mu_n^{(i)}}{\partial \Gamma_n^{(i)}} d\Gamma_n^{(i)} \]

Current:
\[ J_n^{(i)} = -\frac{L_n^{(ii)}}{T} \frac{\partial \mu_n^{(i)}}{\partial \Gamma_n^{(i)}} \]

Chemical potential:
\[ \mu_n^{(i)}(\Gamma_n^{(i)}, t) = RT \ln(\psi_n^{(i)} p_n^{(i)}) + \phi_n^{(i)}(\Gamma_n^{(i)}) \]
First-order structures:

\[
\frac{\partial p_n(\Gamma_n, t)}{\partial t} = \sum_{i=1}^{4} D_i \frac{\partial}{\partial \Gamma_n^{(i)}} \left[ \frac{\partial p_n^{(i)}}{\partial \Gamma_n^{(i)}} + p_n^{(i)} \frac{\partial \ln \psi_n^{(i)}}{\partial \Gamma_n^{(i)}} \right] + \frac{p_n^{(i)}}{k_B T} \frac{\partial \phi_n^{(i)}}{\partial \Gamma_n^{(i)}}
\]

Second-order structures:

\[
\frac{\partial q_n(\Gamma_n', t)}{\partial t} = \sum_{i=1}^{2} D_{s,i} \frac{\partial}{\partial \Gamma_n'^{(i)}} \left[ \frac{\partial q_n^{(i)}}{\partial \Gamma_n'^{(i)}} + q_n^{(i)} \frac{\partial \ln \psi_n'^{(i)}}{\partial \Gamma_n'^{(i)}} \right] \frac{\partial \ln \psi_n'^{(i)}}{\partial \Gamma_n'^{(i)}} + \frac{q_n^{(i)}}{k_B T} \frac{\partial \phi_n'^{(i)}}{\partial \Gamma_n'^{(i)}}
\]
RESULTS

Adding activator

Coarse-graining in the reaction coordinates

(a) DBC [mM]

(b) DBCMe [mM] & DBCMe2 [mM]

(c) Dynamic Light Scattering [counts per second]
Entropy production rate

\[ \sigma(t; \eta) = -\frac{1}{T} \int_{\Gamma} \dot{J}_\Gamma(\Gamma, t; \eta) \cdot \nabla \mu(\Gamma, t; \eta) d\Gamma \]

\[ \sigma(t; \eta) = \sum_j k_j (\Delta_j \mathcal{Z}(t; \eta))^2 \]

\[ \sigma_g(t; N) = \sum_{i=1}^{2} \sigma_{0,i} + \sum_{n=2}^{N} \sum_{i=1}^{4} \sigma_{1,i}^{(n)} + \sum_{n=2}^{\infty} \left[ \sum_{m=2}^{\infty} \sigma_{2,1}^{(n,m)} + \sum_{y=1}^{n-1} \sigma_{2,2}^{(n,n-y)} \right] \]

The total entropy produced increases monotonically but **not linearly**. From experimental results, we observe that this quantity is maximized.
Liesegang rings

❖ Periodic precipitation pattern.
❖ Their control and engineering constitute a crucial task for applications.
❖ Liesegang patterns are composed of mono-disperse nano- and micro-structures whose size varies predictably from ring to ring.
Electrolyte E diffuses into the matrix and interacts with electrolyte B to trigger the bottom-up self-assembly of meso-structures leading to the Liesegang patterns.

- **Structures size** increases as a function of the radial position.
Mechanism

(1) $2E + B \rightleftharpoons A$

(1) First order activation (chemical reaction)

(2) $nA \rightleftharpoons A_n$

(2) First order self-assembly (chemical reaction/pre-nucleation)

(3) $A_n \rightleftharpoons G$

(3) Second order activation (phase change)

(4) $mG + lE \rightarrow G_m$

(4) Second order self-assembly (aggregation)

$$R_1 = k_1^+(T)C_B C_E^2 - k_1^-(T)C_A,$$

$$R_2 = k_2^+(T)C_A^2 - k_2^-(T)C_{A_n},$$

$$R_3 = k_3^+(T)C_{A_n}^2 - k_3^-(T)C_G,$$

$$R_4 = \begin{cases} k_4(T)C_G C_{G_m}^2 & \text{if } C_G < C_{lim} \\ k_4(T)C_G C_{G_m}^2 + k_4(T)C_E C_G (C_G - C_{lim}) & \text{if } C_G \geq C_{lim} \end{cases}$$

E: activator (silver nitrate)
B: disactivated BB (potassium dichromate)
A: activated BB (silver dichromate)
A_n: first order structure
G_m: second order structure
Model


Continuity equation

\[
\frac{\partial p_i(\Gamma, t)}{\partial t} = -\nabla_{\Gamma} \cdot J_i(\Gamma, t),
\]

Energy Balance

\[
\frac{\partial e(r,t)}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} rJ_e(r,t),
\]

Structure size:

\[
d(r,t) = d_0 + \omega \ln\left(\frac{C_E(r_{lim}, 0)}{C_E(r, t)}\right), \quad \forall r > r_{lim},
\]

Experiments:

\[
\frac{\Delta d}{\Delta C_E} \propto -1/C_E
\]
Results

❖ Ring distribution obtained from our model at time $t_p$ (smaller than relaxation time).

❖ Darker color corresponds to the highest concentration of precipitated structures while the lighter color indicates absence of structure.

❖ From the experiments, we estimate $\omega$ and $d_0$. 
We observe a ring distribution in which the separation between the rings increases with the distance.

The diameter of the structure and its standard deviation as a function of position tends to increase.

The error in the estimated position and in the structure's diameter is lower in the non-isothermal model.
The system evolves towards a structure which \textbf{minimizes} the lost work in its formation.

The structure thus adopts an \textbf{architecture} such that the work lost in \textbf{changing the configuration} is almost negligible.
CONCLUSIONS

❖ Models based on **mesoscopic thermodynamics** enable us to understand and to **describe the evolution** of the **internal architecture** of meso-structures.

❖ The model allow us to **compute the entropy production** of the NESA process as a function of the **structural parameter**.

❖ The structures observed in this system are found at **extreme values of the entropy produced** when represented as a function of a **structural parameter**.
First-order structures:
\[
\left. \frac{\partial p_n}{\partial \Gamma_n^{(\gamma)}} \right|_{\Gamma_n=0} = \left. \frac{\partial p_n}{\partial \Gamma_n^{(\gamma')}} \right|_{\Gamma_n=0} = \left. \frac{\partial p_n}{\partial \Gamma_n^{(\lambda)}} \right|_{\Gamma_n=0} = \left. \frac{\partial p_n}{\partial \Gamma_n^{(\theta)}} \right|_{\Gamma_n=0} \\
= J_{n-1}^{(\gamma)}(1,t) + J_{n+1}^{(\gamma')} (1,t) + J_n^{(\lambda)}(\pi,t)
\]

Second-order structures:
\[
\left. \frac{\partial q_n}{\partial \Gamma_n^{(\gamma_n)}} \right|_{\Gamma_n'=0} = \left. \frac{\partial q_n}{\partial \Gamma_n^{(\gamma_n')}} \right|_{\Gamma_n'=0} = \left. \frac{\partial q_n}{\partial \Gamma_n^{(\lambda_n)}} \right|_{\Gamma_n'=0} = \left. \frac{\partial q_n}{\partial \Gamma_n^{(\theta_n')}} \right|_{\Gamma_n'=0} \\
= J_{n}^{(\lambda)}(1,t) + \sum_{m=1}^{[n/2]} J_{m}^{(\gamma_n)}(1,t) + \sum_{y=n+1}^{N} J_{y}^{(\gamma_n')}(1,t)
\]
Modelling potential barriers

\[ \phi(0) = \mu^0_A \]
\[ \phi(1) = \mu^0_B \]
\[ \phi(\gamma^*) = \max(\phi(\gamma)) - \mu^0_A = \epsilon \]

\[ \frac{d\phi(\gamma)}{d\gamma} \bigg|_{\gamma=0} = 0 \]
\[ \frac{d\phi(\gamma)}{d\gamma} \bigg|_{\gamma=1} = 0 \]
\[ \frac{d\phi(\gamma)}{d\gamma} \bigg|_{\gamma=\gamma^*} = 0 \]

\[ \exp\left(\frac{\mu^0_A - \mu^0_B}{RT}\right) = \frac{P_B,eq}{P_{A,eq}} = \frac{\int^{1}_{\gamma^*} \exp\left(-\phi(\gamma)/RT\right) d\gamma}{\int^{\gamma^*}_{0} \exp\left(-\phi(\gamma)/RT\right) d\gamma} \]

\[ \phi(\gamma) = \sum_{i=0}^{5} c_i \gamma^i \]

A: \( \gamma < \gamma^* \)

B: \( \gamma \geq \gamma^* \)

Bistable

\[ \int_{0}^{\gamma_{\text{ref}}} \exp(\phi(\gamma)/RT) d\gamma \approx \exp(E_a/RT) \]
Modelling a potential barrier with an intermediate state

\[ \phi(\gamma) = \begin{cases} 
\phi^-(\gamma) & 0 \leq \gamma \leq \gamma_{ES} \\
\phi^+\left(\gamma\right) & \gamma_{ES} \leq \gamma \leq 1 
\end{cases} \]

\[ \frac{d\phi^-\left(\gamma\right)}{d\gamma} \bigg|_{\gamma=0} = 0 \]
\[ \frac{d\phi^+\left(\gamma\right)}{d\gamma} \bigg|_{\gamma=\gamma_{ES}} = 0 \]
\[ \frac{d\phi^-\left(\gamma\right)}{d\gamma} \bigg|_{\gamma=\gamma^*} = 0 \]

\[ \phi(0) = \mu_S^0 \]
\[ \phi(\gamma_{ES}) = \mu_{ES}^0 \]
\[ \phi(1) = \mu_P^0 \]
\[ \phi\left(\gamma^*_+\right) = \max\left(\phi^-(\gamma)\right) - \mu_S^0 = \epsilon^- \]

\[ \phi\left(\gamma^*_-\right) = \max\left(\phi^+\left(\gamma\right)\right) - \mu_P^0 = \epsilon^+ \]

\[ \exp\left(\left(\mu_S^0 - \mu_{ES}^0\right)/RT\right) = \frac{P_{ES,eq}}{P_{S,eq}} = \frac{\int_{\gamma_{ES}}^{\gamma^*_+} \exp\left(-\phi^-(\gamma)/RT\right)d\gamma + \int_{\gamma_{ES}}^{\gamma^*_-} \exp\left(-\phi^+\left(\gamma\right)/RT\right)d\gamma}{\int_{0}^{\gamma^*_+} \exp\left(-\phi^-\left(\gamma\right)/RT\right)d\gamma} \]

\[ \exp\left(\left(\mu_S^0 - \mu_P^0\right)/RT\right) = \frac{P_{P,eq}}{P_{S,eq}} = \frac{\int_{\gamma_{ES}}^{\gamma^*_+} \exp\left(-\phi^+\left(\gamma\right)/RT\right)d\gamma + \int_{\gamma_{ES}}^{\gamma^*_-} \exp\left(-\phi^-\left(\gamma\right)/RT\right)d\gamma}{\int_{0}^{\gamma^*_+} \exp\left(-\phi^-\left(\gamma\right)/RT\right)d\gamma} \]