PoreLab
NTNU-UiO Porous Media Laboratory

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Energy Budget of Fiber Bundle Model

SK-70, NTNU

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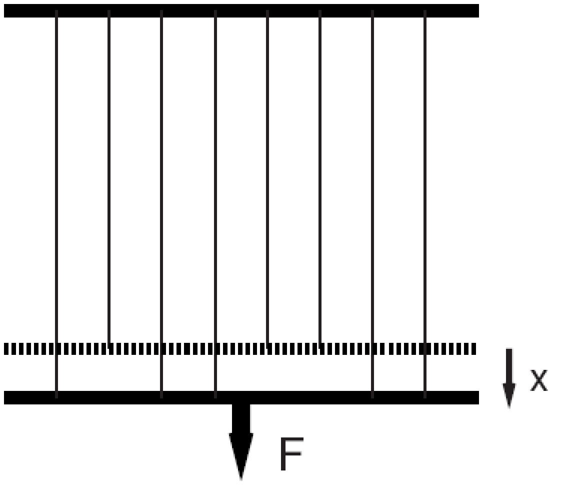
Fiber bundle model (FBM)

- First used in textile engineering (Peirce, 1926)

- Statistical analysis (Daniels,1945)

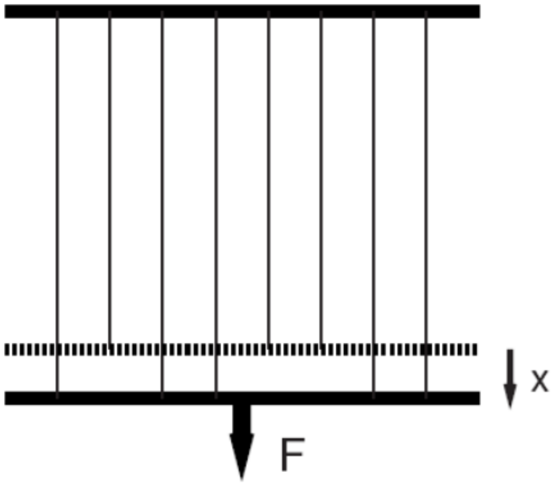
- Different load-sharing rules:

ELS, LLS, mixed-mode, hierarchical



Different ways of Loading

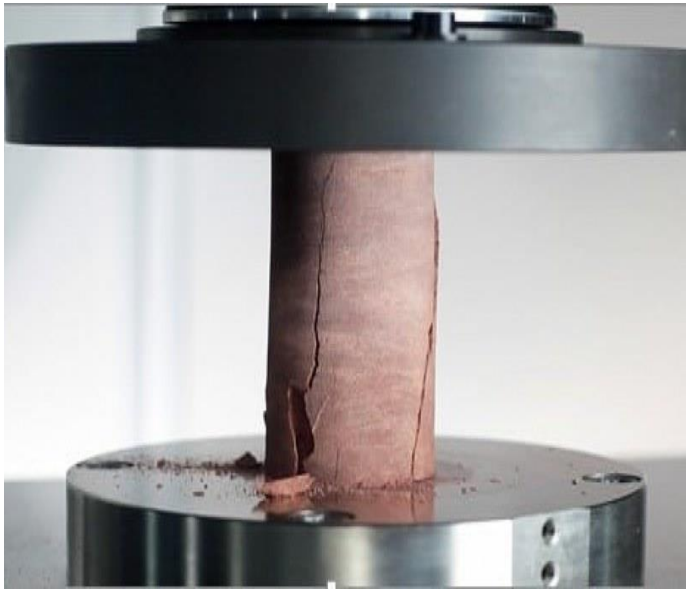
A) Quasi-static loading: weakest link failure



B) Loading by a fixed amount: expt. at lab

C) Overloaded situation: applied load > critical load

Problems in real-life

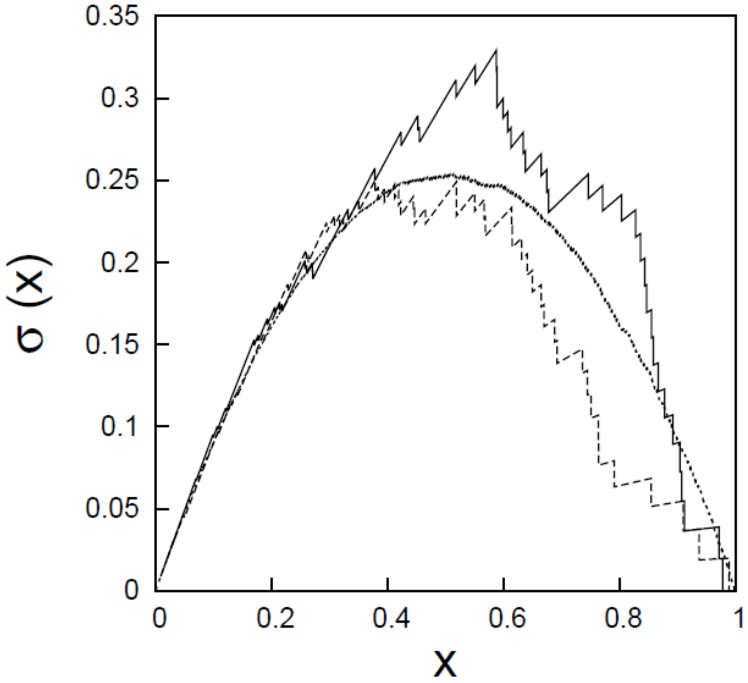


Static: Force-displacement

$$F(x) = N[1 - P(x)]\kappa.x$$

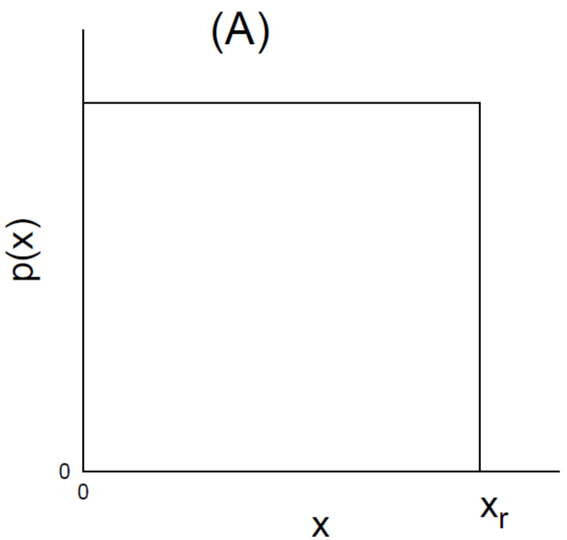
$$P(y) = \int_0^y p(x)dx$$

$$\sigma = F(x)/N = [1 - P(x)]\kappa.x$$



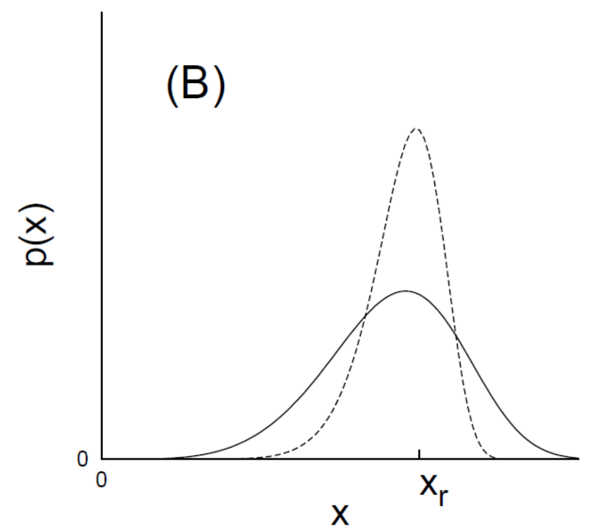
Static: Critical strength

- F(x) has a maximum
- Put $\frac{dF}{dx} = 0$ $$1 - P(x_c) - x_c p(x_c) = 0$$



$$P(x) = x; \quad x_c = \frac{1}{2}$$

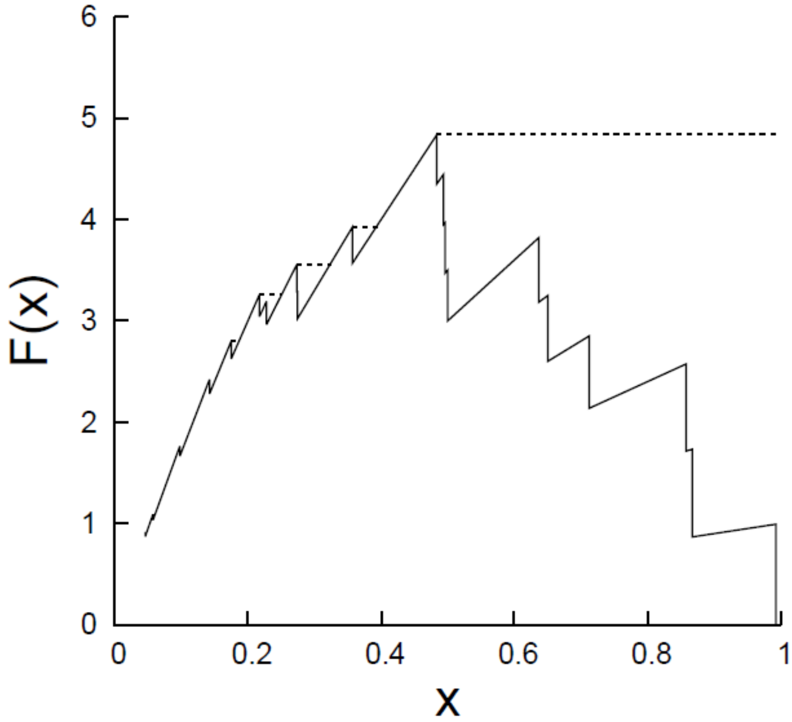
Uniform dist.



$$P(x) = 1 - e^{-x^k}; \quad x_c = k^{-\frac{1}{k}}$$

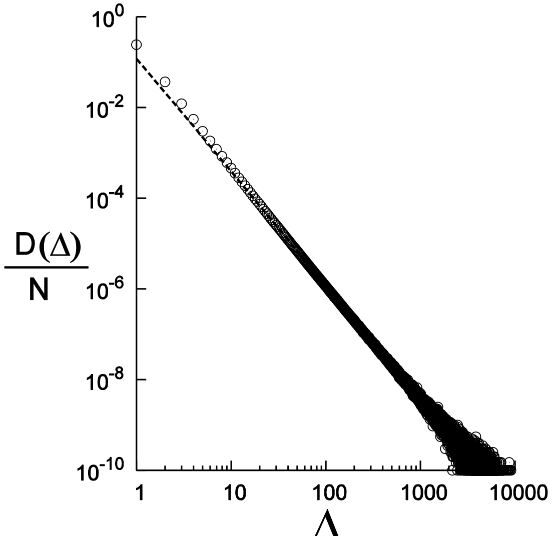
Weibull dist.

Static: Burst or avalanche



$$D(\Delta) \propto \Delta^{-5/2}$$

Hemmer & Hansen, 1992

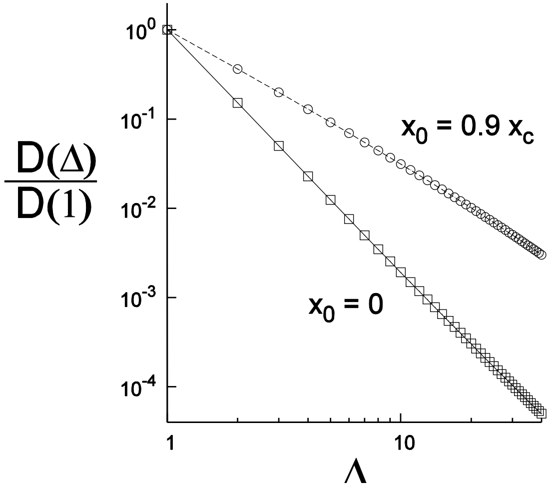


$$N = 10^6; avg = 20000$$

Static: Crossover behavior

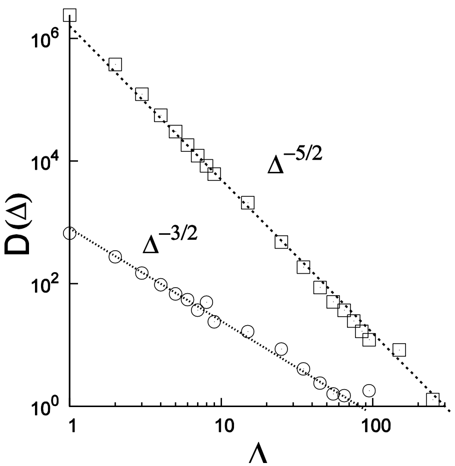
$$D(\Delta) \propto \Delta^{-5/2}(1-e^{-\Delta/\Delta_c})$$

$$\Delta_c = \frac{1}{8(x_c - x_0)^2}$$



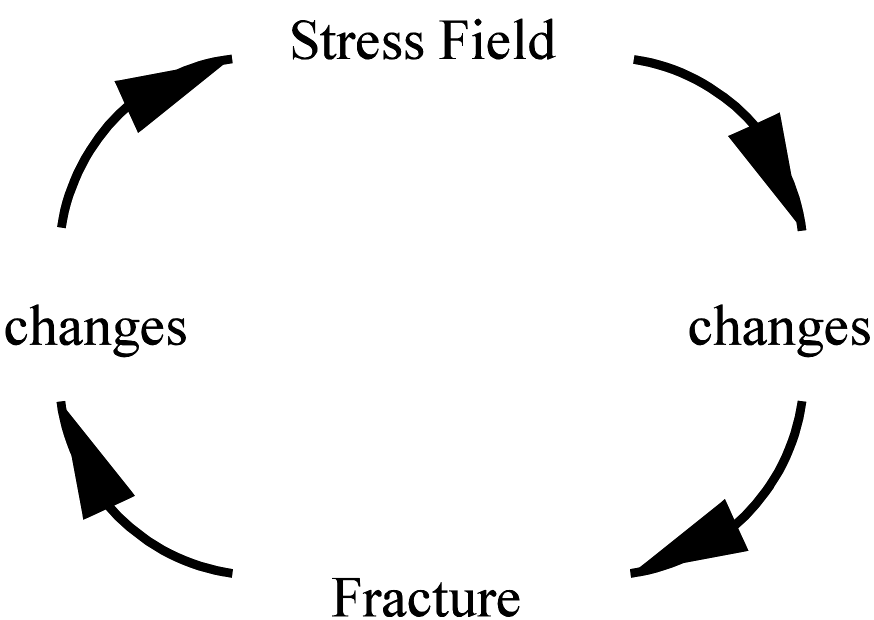
(*Pradhan, Hemmer & Hansen*; *2005*)

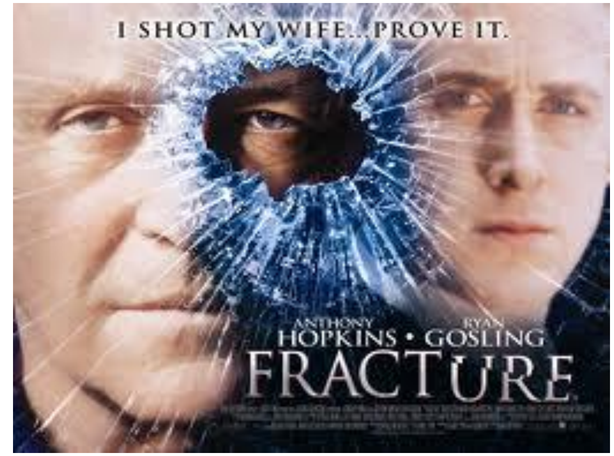
$N = 10^6; avg = 50000$



Single sample $N = 10^7$

Dynamics of fracture





Crack-growth in nano-materials
……onset of earthquake

- The recursive dynamics

$$U_t = N_t / N$$

$$P(y) = \int_0^y p(x)dx$$

$$U_{t+1} = 1 - P(\sigma / U_t)$$

Uniform dist. $$U_{t+1} = 1 - \sigma / U_t$$

Fixed point $$U_{t+1} = U_t = U^* \longrightarrow U^{*2} - U^* + \sigma = 0$$

Solution $$U^*(\sigma) = \frac{1}{2} \pm (\sigma_c - \sigma)^{1/2}$$

(*Pradhan, Bhattacharyya & Chakrabarti*; *2001-02*)

Dynamic: Critical behavior

Order parameter

$$O = U^*(\sigma) - U^*(\sigma_c) \approx (\sigma_c - \sigma)^{1/2}$$

Susceptibility

$$\chi = \frac{dU^*(\sigma)}{d\sigma} \approx (\sigma_c - \sigma)^{-1/2}$$

Differential form

$$\frac{dU}{dt} = U_t - U_{t+1} = U_t - 1 + \frac{\sigma}{U_t}$$

Relaxation time

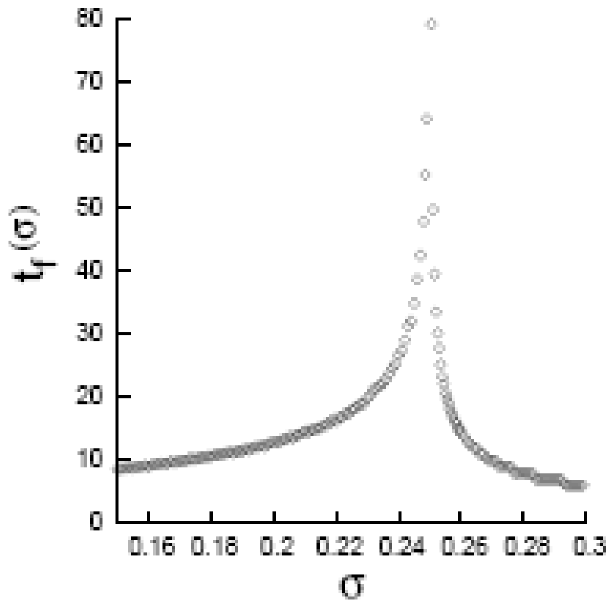
$$\tau \propto (\sigma_c - \sigma)^{-1/2}$$

(*Pradhan, Bhattacharyya & Chakrabarti*; *2001-02*)

Dynamic: Failure step/time (Simulation)

Above critical stress

$t_f(\sigma)$ is the failure step/time



Below critical stress

$t_f(\sigma)$ is the step/time to reach the fixed point

Exact solution: Above critical stress

$$U_{t+1} = 1 - \frac{\sigma}{U_t} = 1 - \frac{\frac{1}{4} + \varepsilon}{U_t}$$

$$U_t = \tfrac{1}{2} - y_t \sqrt{\varepsilon} \quad \Longrightarrow \quad \frac{y_{t+1} - y_t}{1 + y_t y_{t+1}} = 2\sqrt{\varepsilon}$$

$$y_t = \tan v_t \quad \Longrightarrow \quad 2\sqrt{\varepsilon} = \frac{\tan v_{t+1} - \tan v_t}{1 + \tan v_{t+1} \tan v_t} = \tan(v_{t+1} - v_t)$$

$$v_t = v_0 + t \tan^{-1}(2\sqrt{\varepsilon})$$

solution

$$t_f(\sigma) = \tfrac{\pi}{2}(\sigma - \sigma_c)^{-1/2}$$

Pradhan & Hemmer 2006

Exact Solution: Below critical stress

$$U_{t+1} = 1 - \frac{\sigma}{U_t} = 1 - \frac{\frac{1}{4} - \varepsilon}{U_t}$$

$$U_t = \frac{1}{2} + \frac{\sqrt{\varepsilon}}{z_t} \quad \Longrightarrow \quad \frac{z_{t+1} - z_t}{1 - z_t z_{t+1}} = 2\sqrt{\varepsilon}$$

$$z_t = \tanh w_t \quad \Longrightarrow \quad 2\sqrt{\varepsilon} = \frac{\tanh w_{t+1} - \tanh w_t}{1 - \tanh w_{t+1} \tanh w_t} = \tanh(w_{t+1} - w_t)$$

$$w_t = w_0 + t \tanh^{-1}(2\sqrt{\varepsilon})$$

solution

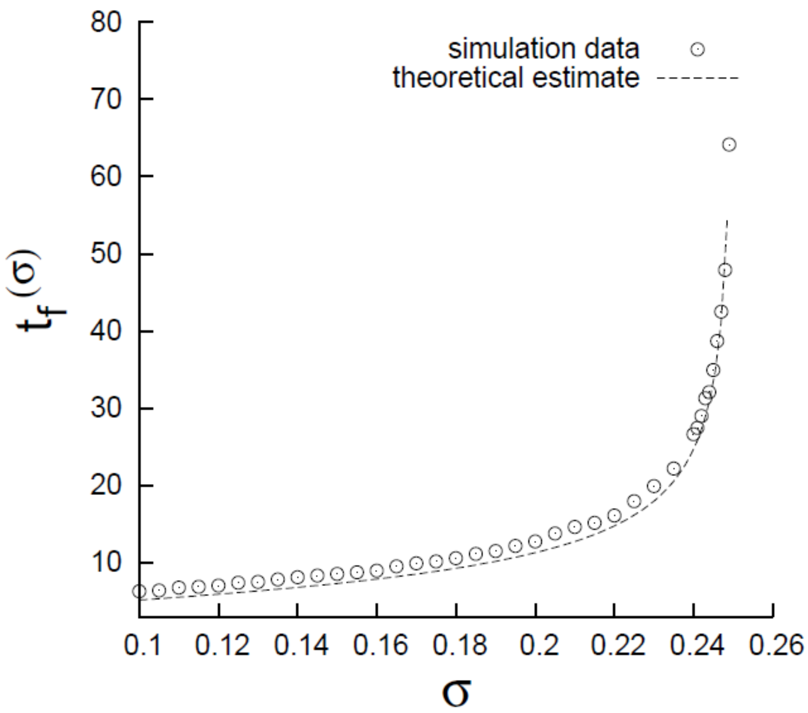
$$t_f(\sigma) = \frac{\ln(N)}{4}(\sigma_c - \sigma)^{-1/2}$$

Critical amp. ratio $\frac{\ln(N)}{2\pi}$

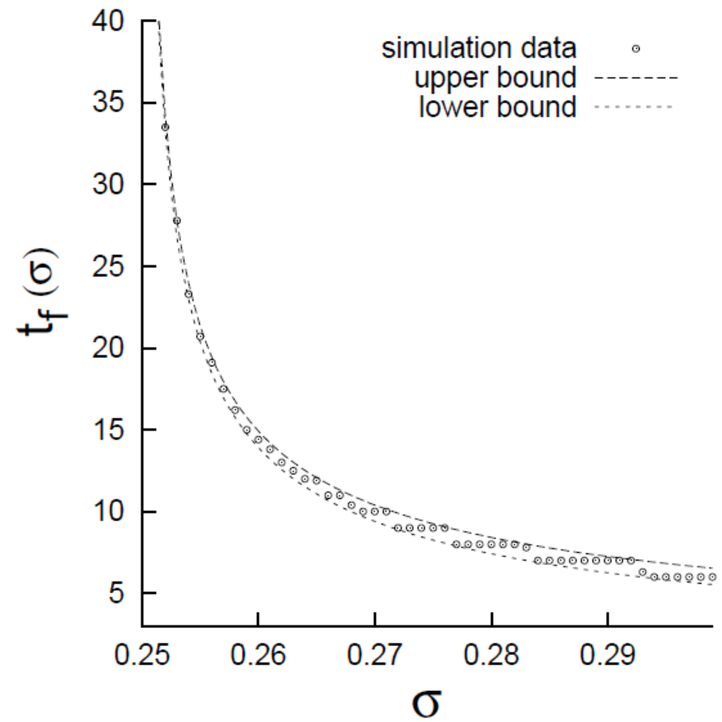
Pradhan & Hemmer 2006

Theory vs. Simulation

Below critical stress



Above critical stress

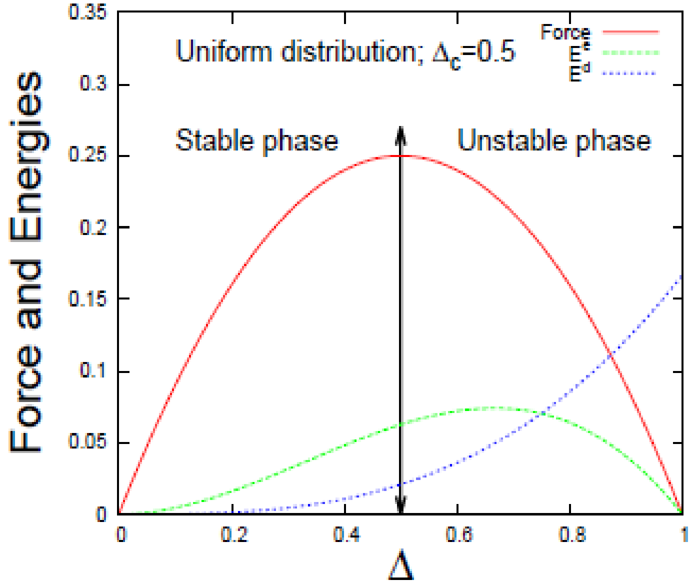


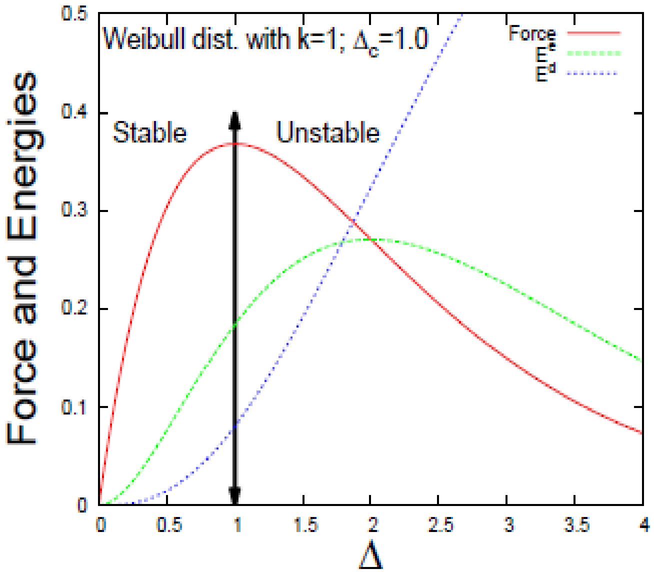
Pradhan & Hemmer 2006

Energy Budget of FBM

$$E^e(\Delta) = \frac{Nk}{2}\Delta^2\ (1\text{-}P(\Delta))$$

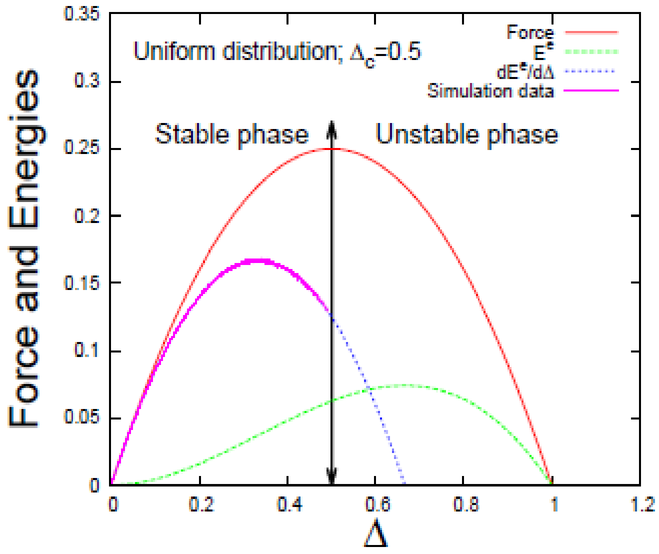
$$E^d(\Delta) = \frac{Nk}{2}\int_0^{\Delta} x^2 p(x)dx$$

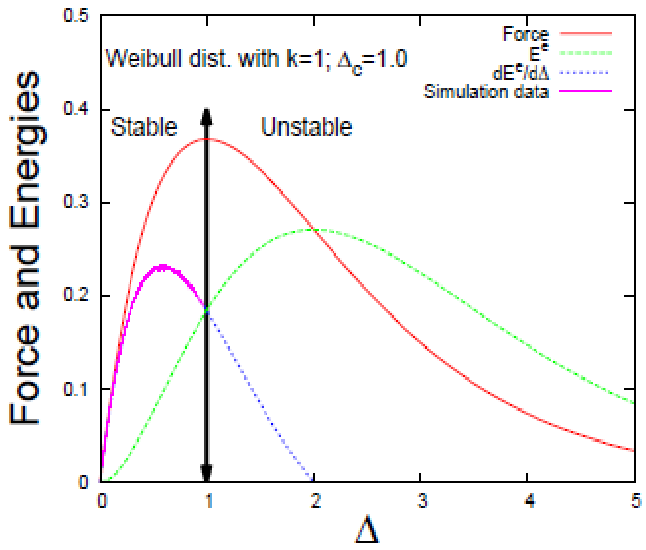




Energy Budget: Signal of upcoming failure

$$\frac{dE^e}{d\Delta}$$ has a maximum in the stable phase

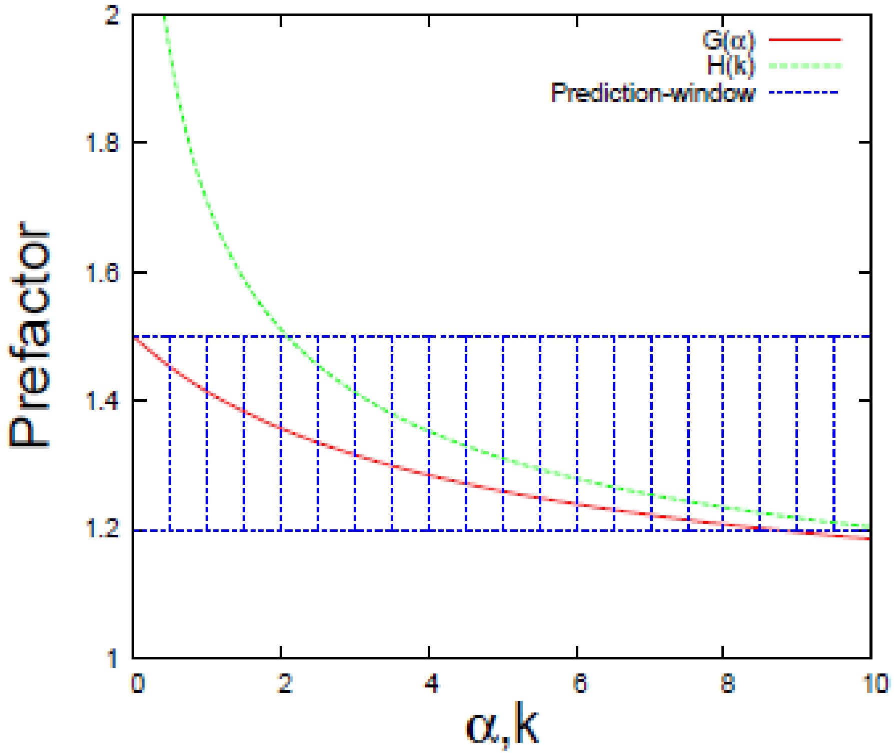


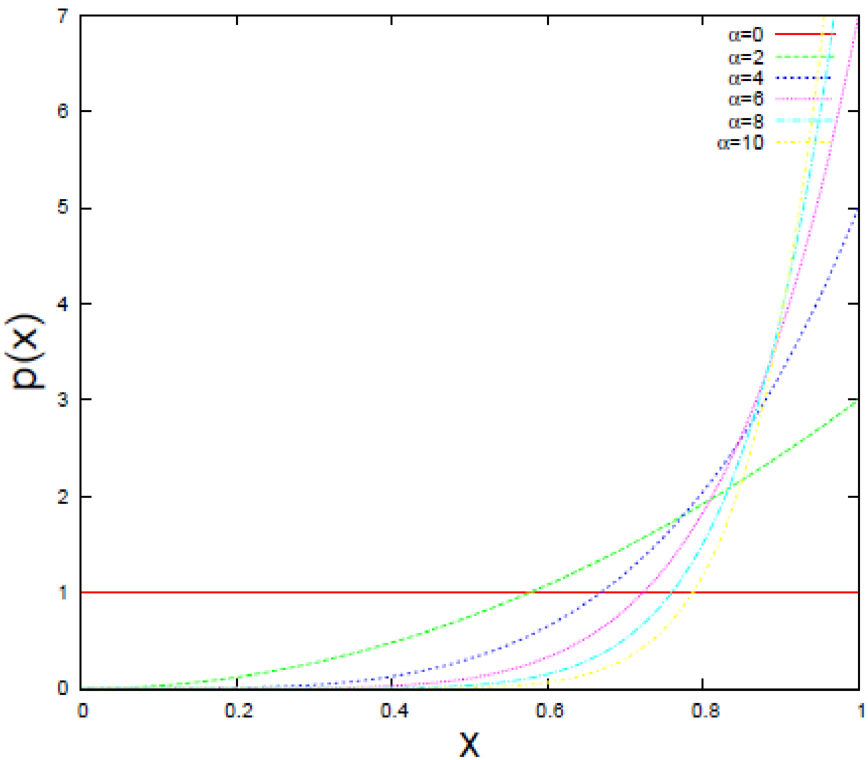


Energy Budget: Signal of upcoming failure

$$\Delta_c = G(\alpha)\Delta_{max}$$

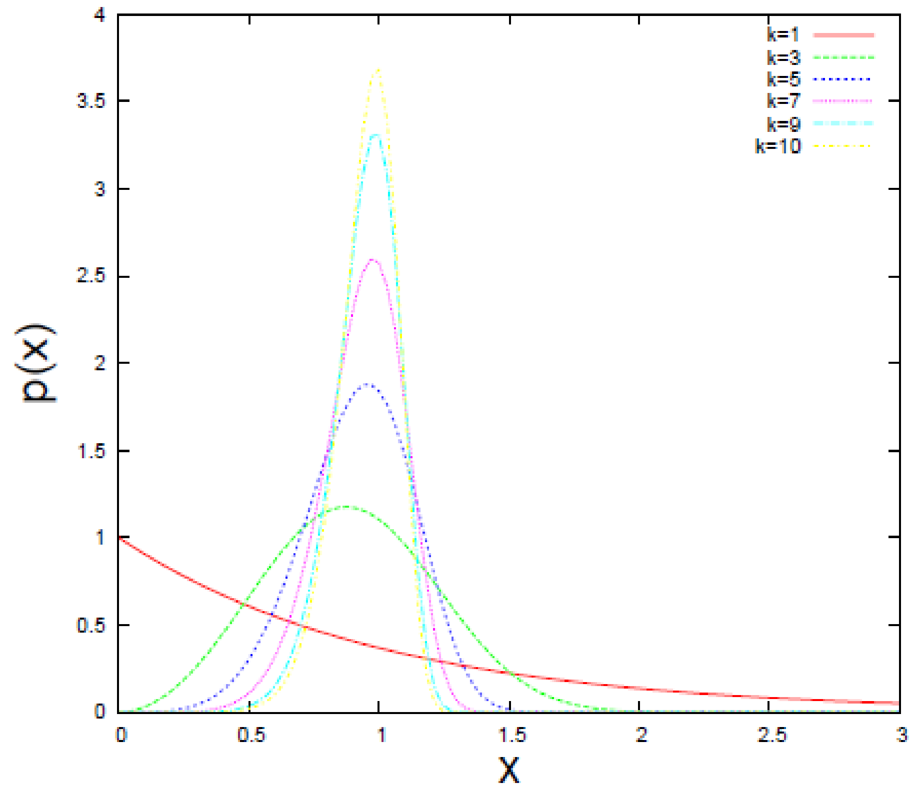
$$\Delta_c = H(k)\Delta_{max}$$





Power-law type dist.

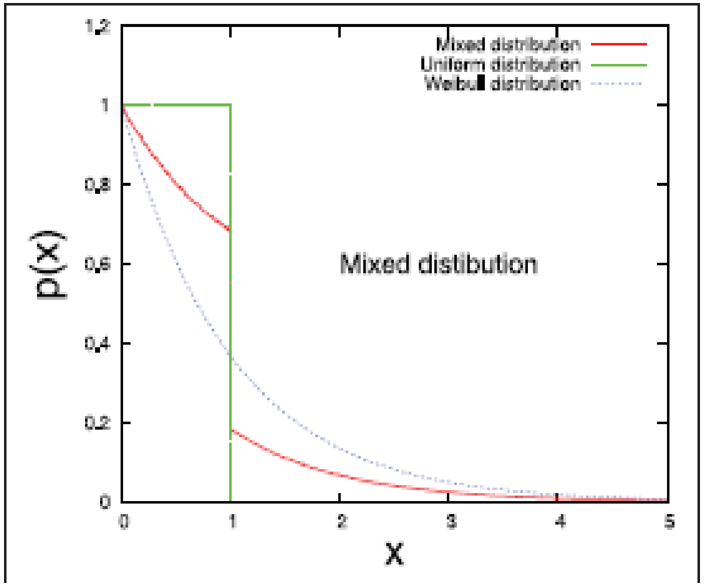
$$P(x) = x^{\alpha+1}$$

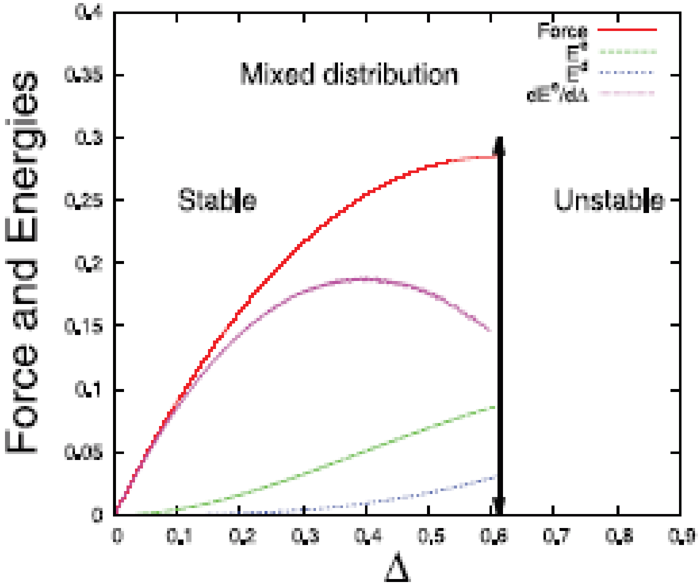


Weibull dist.

$$P(x) = 1 - e^{-x^k}$$

Test-case: Mixed-distribution





Conclusions

- Elastic energy growth rate shows a peak before collapse point.
- Signal of upcoming failure can be used to predict collapse-situations (rock failure, mine-collapse, borehole collapse …)
- The energy framework will help to formulate a thermodynamic description of FBM
- Can we calculate/check minimum entropy production principle?
- Collaborators: Alex Hansen, Per C. Hemmer, Jonas T. Kjellstadli, Bikas K. Chakrabarti

FBM research at PoreLab

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- Field theory for ELS dynamics (Hendrick, Pradhan, Hansen, PRE 2018)
- RG scheme for ELS models (Pradhan, Ray, Hansen, Front. Phys. 2018)
- Energy budget in FBM (Pradhan, Kjellstadli, Hansen, Front. Phys. 2019)
- Stability issue in LLS models (Jonas, Eivind, .. Front. Phys. 2019)
- Fracture front propagation during fluid injection (Pradhan et al.)
- Stretching of biological systems (Eivind et. al.)
- Flow in a Fiber Tube Model (Roy, Sinha, Hansen, Front. Phys. 2019)