Compressibility-induced destabilisation of falling liquid films: an integral approach

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Abstract. We revisit the classical 2D problem of a gravity-driven liquid layer down an inclined plate (Kapitza, 1948), relaxing the usual assumption of homogeneous fluid. We set out to answer three major issues. When the fluid density is allowed to vary, (i) how does this feature structurally affect the formulation of a low-dimensional depth-averaged model? (ii) To what extent and (iii) by virtue of which physical mechanism does compressibility participate in the long-wave interfacial instability? To provide the relevant answers, (i) we first make use of a second-order asymptotic expansion in the shallowness parameter to develop a weakly-compressible boundary-layer system: starting from a two-equation momentum-integrated model, an additional barotropic equation of state is required for closure purposes. In this respect, (ii) a temporal linear stability analysis is performed: it is revealed that compressibility plays a destabilising role whose magnitude is enhanced at intermediately tilted configurations, and the more the Reynolds number approaches the critical threshold in the incompressible limit. (iii) We finally interpret the ensuing dispersion relation under the convenient framework of two-wave hierarchy, initiated by Whitham (1974): the primary instability gets promoted by the flow compressibility as it contributes to deceleration of dynamic waves most significantly in the low-inertia regime. Indeed, compressibility locally acts as a further boost to the inertia-based mechanism of Kapitza instability by amplifying flow-rate variations within the liquid film.

Keywords: *falling liquid films, interfacial instability, low-dimensional models*

Introduction

Liquid films gliding down an incline are central to numerous areas of pure and applied sciences. They are of practical relevance in many industrial devices and environmental systems, ranging from small–scale reactors to subsurface flows. In CO_2 capture and storage (CCS) chain, the supercritical phase is employed during pipeline transportation for efficiency purposes: it has a liquid–like density, but a gas–like viscosity and compressibility.

Analytical Methods

We therefore question the typical hypothesis of fluid homogeneity in the context of shallow–water equations, retracing the footsteps of a power–series perturbation expansion in the film aspect ratio $\varepsilon \ll 1$: when the definition of isentropic speed of sound is incorporated as constitutive law, the effects of a weak compressibility can be captured by means of a global Sarrau–Mach number $\mathrm{Ma} = \mathcal{O}(\varepsilon^{\alpha})$, with $\alpha = 1$.

Results and Discussion

Following Whitham (1974), when the wavenumber $k \to 0$, an equality criterion between the celerity of competing kinematic and dynamic waves yields a novel neutral stability condition in terms of the critical Reynolds number Re_{cr}, as **Fig. 1** shows.

To investigate the impact of the slope steepness, the ratio $\mathrm{RCR} = \mathrm{Re}_{cr}/\mathrm{Re}_{cr}^{\mathrm{Ma}\to 0+}$ has been employed as a measure of the Ma–induced destabilisation. Keeping it fixed, instability quickly saturates with inclination.

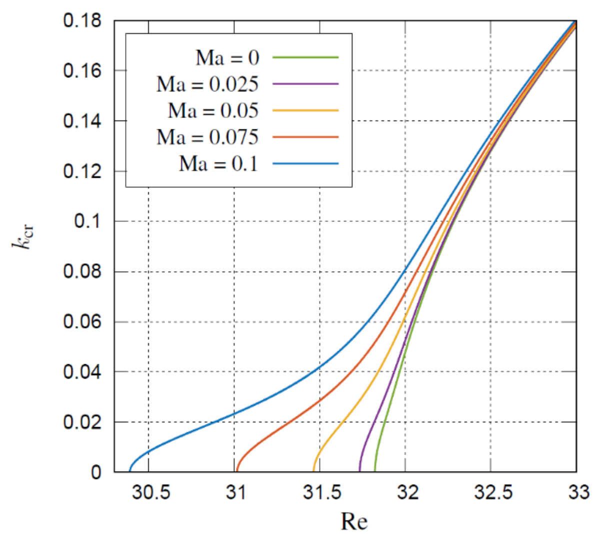

Figure 1: Marginal curves of a falling water–glycerine mixture for different values of Ma – here the wall inclination angle is $\beta = 1.5^{\circ}$.

Conclusions

(i) A small additive flow rate contribution, hydrostatic in nature, is established by a low–degree compressibility.
(ii) According to our parametric study, this boosts the inception of interfacial instability, especially close to the critical threshold in the incompressible limit.
(iii) As physical foundation, compressibility amplifies the inertial flow rate delay in adapting to film–thickness variations.

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